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EIGEN VALUES AND EIGEN VECTORS FOR FUZZY MATRIX

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Abstract- Many applications of matrices in both engineering and science utilize Eigen values and Eigen vectors. Control theory, vibration analysis, electric circuits, advanced dynamics and quantum mechanics are the few of the applications area. In this paper, first time we introduced the Eigen values and eigen vectors of fuzzy matrix. This paper consist four sections. In first section, we give the introduction about Eigen values, Eigen vectors and fuzzy matrix. Proposed definitions of Eigen values and eigen vectors were derived in second section. In the third section, we give the application of proposed Eigen values and Eigen vectors of fuzzy matrix. Conclusions were given in final section.

Keywords- Characteristic Equation, Eigen values, Eigen Vectors and Fuzzy Matrix.

1. INTRODUCTION

The eigen value problem is a problem of considerable theoretical interest and wide-ranging application. For example, this problem is crucial in solving systems of differential equations, analyzing population growth models, and calculating powers of matrices (in order to define the exponential matrix). Other areas such as physics, sociology, biology, economics and statistics have focused considerable attention on “Eigen values” and Eigen vectors”-their applications and their computations.

The basic concept of the fuzzy matrix theory is very simple and can be applied to social and natural situations. A branch of fuzzy matrix theory uses algorithms and algebra to analyze date. It is used by social scientists to analyze interaction between actors and can be used to complement analyses carried out using game theory or other analytical tools.

2. PROPOSED DEFINITIONS AND EXAMPLES

In this section we give the proposed Characteristic Equations of Fuzzy matrix, Polynomial equations of fuzzy matrix, working rule to find characteristic equation of fuzzy matrix, Fuzzy Eigen Values and Eigen vectors, Properties of Fuzzy Eigen values and Eigen vectors are presented as follows:

2.1. Characteristic Equation of Fuzzy Matrix

Consider the linear transformation $Y = A_F X$

In general, this transformation transforms a column vector $X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix}$ into the another column vector $Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ y_n \end{bmatrix}$

By means of the square fuzzy matrix A_F where

$$A_F = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

If a vector X is transformed into a scalar multiple of the same vector. i.e., X is transformed into λX , then $Y = \lambda X = A_F X$

i.e., where I is the unit matrix of order 'n'.

$$A_F X - \lambda I X = O$$

$$(A_F - \lambda I)X = O \qquad \ldots (2.1)$$

$$\left[\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ . & . & \cdots & \cdots \\ . & . & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$\begin{aligned} &(a_{11}-\lambda)x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ &a_{21}x_1 + (a_{22}-\lambda)x_2 + \ldots + a_{2n}x_n = 0 \\ \text{i.e.,}\quad &. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \ldots (2.2) \\ &. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ &a_{n1}x_1 + a_{n2}x_2 + \ldots + (a_{nn}-\lambda)x_n = 0 \end{aligned}$$

This system of equations will have a non-trivial solution, if $|A_F - \lambda I| = 0$

$$\text{i.e.,} \begin{vmatrix} a_{11}-\lambda & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-\lambda & \ldots & a_{2n} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ a_{n1} & a_{n2} & \ldots & a_{nn}-\lambda \end{vmatrix} = 0 \qquad \ldots (2.3)$$

The equation $|A_F - \lambda I| = 0$ or equation (2.3) is said to be the characteristic equation of the transformation or the characteristic equation of the matrix *A*. Solving $|A_F - \lambda I| = 0$, we get *n* roots for λ , these roots are called the characteristic roots (or) Eigen values of the matrix A_F. Corresponding to each of value of λ , the equation $A_F X = \lambda X$ has a non-zero solution vector *X*. Let X_r , be the non-zero vector satisfying $A_F X = \lambda X$. When $\lambda = \lambda_r$, X_r is said to be the latent vector or Eigen vector of a matrix A_F corresponding to λ_r .

2.1.1. Characteristic polynomial of Fuzzy Matrix

The determinant $|A_F - \lambda I|$ when expanded will give a polynomial, which we call as characteristic polynomial of fuzzy matrix A_F.

2.2. Eigen Values and Eigen Vectors of a Fuzzy Matrix

2.2.1. Fuzzy eigen values or Proper values or Latent roots or Characteristic roots

Let $A_F = \left[a_{ij}\right]$ be a square matrix.

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The roots of the characteristic equation are called Fuzzy Eigen values of A_F.

2.2.2. Eigen vectors or Latent vector

Let $A_F = \left[a_{ij}\right]$ be a fuzzy square matrix. If there exists a non-zero vector $X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix}$.

Such that $A_F X = \lambda X$, then the vector X is called Eigenvector of A_F corresponding to the fuzzy eigenvalue λ.

Note:

(i) Corresponding to *n* distinct Fuzzy Eigen values, we get *n* independent Eigen vectors.

(ii) If two or more Fuzzy Eigen values are equal, then it may or may not be possible to get linearly independent Eigenvectors corresponding the repeated Fuzzy Eigen values.

(iii) If X_i is a solution for an Eigen value λ_i, then it follows from $\left(A_F - \lambda I\right)X = O$ that $C\ X_i$ is also a solution, where *C* is an arbitrary constant. Thus, the Eigenvector corresponding to a Fuzzy Eigen value is not unique but may be any one of the vectors *CX*.

(iv) Algebraic multiplicity of an Fuzzy eigenvalue λ is the order of the fuzzy Eigen value as a root of the characteristic polynomial (i.e., if λ is a double root then algebraic multiplicity is 2)

(v) Geometric multiplicity of λ is the number of linearly independent eigenvectors corresponding to λ.

2.2.3. Working rule to find Eigenvalues and Eigenvectors

Step 1: Find the characteristic equation $|A_F - \lambda I| = 0$.

Step 2: Solving the characteristic equation, we get characteristic roots. They are called Fuzzy Eigen values.

Step 3: To find Eigenvectors, solve $\left(A_F - \lambda I\right)X = O$ for the different values of λ.

2.2.4. Non-symmetric matrix

If a fuzzy square matrix A_F is non-symmetric, then $A_F \neq A_F{}^T$.

Note:

(i) In a non-symmetric fuzzy matrix, the Fuzzy Eigen values are non-repeated then we get linearly independent sets of Eigen vectors.

(ii) In a non-symmetric fuzzy matrix the Fuzzy Eigen values are repeated and then we may or may not be possible to get linearly independent eigenvectors. If we form linearly independent sets of eigenvectors, then diagonalisation is possible through similarly transformation.

2.2.5. Symmetric matrix

If a fuzzy square matrix A_F is symmetric, then $A_F = A_F{}^T$

Note:

(i) In a symmetric fuzzy matrix the Fuzzy Eigen values are non-repeated, and then we get a linearly independent and pair wise orthogonal sets of eigenvectors.

(ii) In a symmetric fuzzy matrix the Fuzzy Eigen values are repeated, then we may or may not be possible to get linearly independent and pairwise orthogonal sets of eigenvectors. If we form linearly independent and pairwise orthogonal sets of eigenvectors, the diagonalisation is possible through orthogonal transformation.

2.2.6. Properties of Eigenvalues and Eigenvectors of Fuzzy Matrix

Property 1:

(i) The sum of the Fuzzy Eigenvalues of a matrix is the sum of the elements of the principal (main) diagonal of the Fuzzy Matrix. (or) The sum of the Fuzzy Eigenvalues of a matrix is equal to the trace of the Fuzzy matrix.

(ii) Product of the Fuzzy Eigenvalues is equal to the determinant of the Fuzzy matrix.

Proof: Let A_F be a fuzzy square matrix of order n.

The characteristic equation of A_F is $\left|A_F - \lambda I\right| = 0$

i.e., $\lambda_n - S_1\lambda^{n-1} + S_2\lambda^{n-1} - \ldots + (-1)^n S_n = 0$ … (2.4)

where S_1 =Sum of the diagonal elements of A_F

………

………

S_n = determinant of A_F.

We know the roots of the characteristic equations are called Fuzzy Eigen values of the given fuzzy matrix.

Solving (1) we get n roots.

Let the n roots be $\lambda_1, \lambda_2, \ldots \lambda_n$.

i.e., $\lambda_1, \lambda_2, \ldots \lambda_n$ are the Fuzzy Eigen values of A_F

we know already,

λ^n -(sum of the roots) λ^{n-1} +(sum of the product of the roots taken two at a time) λ^{n-2} - . . . + $(-1)^n$ (Product of the roots) = 0 … (2.5)

Sum of the roots = S_1 by (2.4) and (2.5)

i.e., $\lambda_1 + \lambda_2 + \ldots + \lambda_n = S_1$

i.e., $\lambda_1 + \lambda_2 + \ldots + \lambda_n = a_{11} + a_{22} + ... + a_{nn}$

i.e., Sum of the Fuzzy Eigen values = Sum of the main diagonal elements

Product of the roots = S_n by (2.4) & (2.5)

$\lambda_1, \lambda_2, \ldots \lambda_n$ = det of A_F

i.e., Product of the Fuzzy Eigen values = $|A_F|$

Property: 2

A fuzzy square matrix A_F and its transpose A_F^T have the same Fuzzy Eigen values. (or) A fuzzy square matrix A_F and its transpose A_F^T have the same characteristics values.

Proof: Let A_F be a fuzzy square matrix of order n .

The characteristic equation of A_F and A_F^T are $|A_F - \lambda I| = 0$ … (2.6)

and $|A_F^T - \lambda I| = 0$ … (2.7)

Since the determinant value is unaltered by the interchange of rows and columns.

We know $|A| = |A^T|$

Hence, (1) and (2) are identical.

Therefore, Fuzzy Eigen values of A_F and A_F^T is the same.

Note: A determinant remains unchanged when rows are changed into columns and columns into rows.

Property: 3

The characteristic roots of a triangular fuzzy matrix are just the diagonal elements of the fuzzy matrix. (or) The Fuzzy Eigenvalues of a triangular fuzzy matrix are just the diagonal elements of the fuzzy matrix.

Proof: Let us consider the triangular fuzzy matrix.

$$A_F = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Characteristic equation of A_F is $|A_F - \lambda I| = 0$

i.e., $\begin{vmatrix} a_{11}-\lambda & 0 & 0 \\ a_{21} & a_{22}-\lambda & 0 \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$

On expansion it gives $(a_{11}-\lambda)(a_{22}-\lambda)(a_{33}-\lambda) = 0$

i.e., $\lambda = a_{11}, a_{22}, a_{33}$

which are diagonal elements of fuzzy matrix A_F.

Property: 4

Prove that if λ is an Fuzzy Eigen value of a fuzzy matrix A_F, then $\frac{1}{\lambda}, (\lambda \neq 0)$ is the Eigenvalue of A_F^{-1}.

Proof: If X be the Eigen vector corresponding to λ, then $A_F X = \lambda X$ … (2.9)

Pre multiplying both sides by A_F^{-1}, we get

$$A_F^{-1} A X = A_F^{-1} \lambda X$$

$$IX = \lambda A_F^{-1} X$$

$$X = \lambda A_F^{-1} X$$

$$\div \lambda \Rightarrow \frac{1}{\lambda} X = A_F^{-1} X$$

$$A_F^{-1} X = \frac{1}{\lambda} X$$

This being of the same form as (i), shows that $\frac{1}{\lambda}$ is an Fuzzy Eigen values of the inverse matrix A_F^{-1}.

Property: 5

Prove that if λ is a Fuzzy Eigen value of an orthogonal fuzzy matrix, and then $\frac{1}{\lambda}$ is also Fuzzy Eigen value.

Proof:

By the definition of orthogonal fuzzy matrix

A Fuzzy square matrix A_F is said to be orthogonal if $A_F A_F^T = A_F^T A_F = I$

i.e., $A_F^T = A_F^{-1}$

Let A_F^{-1} be an orthogonal fuzzy matrix

Given λ is a Fuzzy Eigen value of A_F

$\Rightarrow \frac{1}{\lambda}$ is and Fuzzy Eigen value of A_F^{-1}.

Since, $A_F^T = A_F^{-1}$

Therefore, $\frac{1}{\lambda}$ is a Fuzzy Eigen value of A_F^T.

But, the matrices A_F and A_F^T have the same Fuzzy Eigen values, since the determinants $|A_F - \lambda I|$ and $|A_F^T - \lambda I|$ are the same.

Hence $\frac{1}{\lambda}$ is also a Fuzzy Eigen value of A_F

Property: 6

Prove that if $\lambda_1, \lambda_2, \ldots \lambda_n$ are the fuzzy Eigen values of a fuzzy matrix A_F, then A_F^m has the Fuzzy Eigen values $\lambda_1^m, \lambda_2^m, \ldots, \lambda_n^m$, ($m$ being a positive integer)

Proof:

Let A_{F_i} be the fuzzy eigen values of A_F and X_i the corresponding Eigen vector.

Then, $A_F X_i = \lambda_i X_i$ … (2.10)

We have $A_F^2 X_i = A_F\ (A_F X_i)$

$$= A_F\ (\lambda_i X_i)$$
$$= \lambda_i\ A_F (X_i)$$
$$= \lambda_i\ (\lambda_i X_i)$$
$$= \lambda_i^2 X_i$$

|||ly $A_F^3 X_i = \lambda_i^3 X_i$

In general, $A_F^m X_i = \lambda_i^m X_i$ … (2.11)

(2.10) and (2.11) are in same form.

Hence λ_i^m is a fuzzy eigenvalue of A_F^m.

The corresponding Eigenvector is the same X_i.

Note : If λ is the Eigenvalue of the matrix A_F then λ^2 is the Eigenvalue of A_F^2.

Property: 7

The fuzzy eigen values of a fuzzy symmetric matrix are fuzzy numbers.

Proof :

Let λ be an Fuzzy Eigenvalue (may be complex) of the fuzzy symmetric matrix A_F. Let the corresponding Eigenvector be *X*, Let $A_F\,'$ denote the transpose of A_F.

We have $A_F X = \lambda X$

Pre-multiplying this equation by $1\times n$ matrix $\bar{X}\,'$, where the bars denotes that all elements of $\bar{X}\,'$ are the complex conjugate of those of $X\,'$, we get

$$\bar{X}\,' A_F X = \lambda \bar{X}\,' X \quad \text{… (2.12)}$$

Taking the conjugate complex of this we get $X\,' \bar{A}_F \bar{X} = \bar{\lambda} X\,' \bar{X}$ of

$$X\,' A_F \bar{X} = \bar{\lambda} X\,' \bar{X}$$

since $\bar{A}_F = A_F$ for A_F is real.

Taking the transpose on both sides, we get

$$\left(X\,' A_F \bar{X}\right)' = \left(\bar{\lambda} X\,' \bar{X}\right) \text{ (i.e.,) } \bar{X}\,' A_F\,' X = \bar{\lambda} \bar{X}\,' X$$

(i.e.,) $\bar{X}' A_F X = \bar{\lambda} \bar{X}' X$ since $A_F' = A_F$ for A_F is symmetric.

But from (1), $\bar{X}' A_F X = \bar{\lambda} \bar{X}' X$ hence $\lambda \bar{X}' X = \bar{\lambda} \bar{X}' X$

Since $\bar{X}' X$ is an 1×1 matrix whose only element is a positive value, $\lambda = \bar{\lambda}$ (i.e.,) λ is real.

Property 8:

The Eigenvectors corresponding to distinct fuzzy eigen values of a fuzzy symmetric matrix are orthogonal.

Proof:

For a fuzzy symmetric matrix A_F, the Eigen values are fuzzy.

Let X_1, X_2 be Eigenvectors corresponding to two distinct fuzzy eigen values λ_1, λ_2 [λ_1, λ_2 are fuzzy numbers]

$$A_F X_1 = \lambda_1 X_1 \quad \ldots (2.13)$$

$$A_F X_2 = \lambda_2 X_2 \quad \ldots (2.14)$$

Pre multiplying (2.13) by X_2', we get

$$X_2' A_F X_1 = X_2' \lambda_1 X_1$$

$$= \lambda_1 X_2' X_1 \quad \ldots (2.15)$$

Pre-multiplying (2.14) by X_1', we get

$$X_1' A_F X_2 = \lambda_2 X_1' X_2$$

But $(X_2' A_F X_1)' = (\lambda_1 X_2' X_1)'$

$$X_1' A_F' X_2 = \lambda_1 X_1' X_2$$

(i.e.,) $X_1' A_F X_2 = \lambda_1 X_1' X_2$ … (2.16)

From (2.15) and (2.16)

$$\lambda_1 X_1' X_2 = \lambda_2 X_1' X_2$$

(i.e.,) $(\lambda_1 - \lambda_2) X_1' X_2 = O$

$$\lambda_1 \neq \lambda_2, \; X_1' X_2 = O$$

$\therefore X_1 X_2$ are orthogonal.

Property 9:

The similar matrices have same fuzzy eigen values.

Proof:

Let A_F, B_F be two similar fuzzy matrices.

Then, there exists a non-singular fuzzy matrix *P* such that $B_F = P^{-1} A_F P$

$$\begin{aligned} B_F - \lambda I &= P^{-1} A_F P - \lambda I \\ &= P^{-1} A_F P - P^{-1} \lambda I P \\ &= P^{-1} (A_F - \lambda I) P \end{aligned}$$

$$\begin{aligned} | B_F - \lambda I | &= | P^{-1} | \, | A_F - \lambda I | \, | P | \\ &= | A_F - \lambda I | \, | P^{-1} P | \\ &= | A_F - \lambda I | \, | I | \\ &= | A_F - \lambda I | \end{aligned}$$

Therefore, A_F, B_F have the same characteristic polynomial and hence characteristic roots.

They have same fuzzy eigen values.

Property 10:

If a fuzzy symmetric matrix of order 2 has equal fuzzy eigen values, then the matrix is a scalar matrix.

Proof:

Rule 1: *A* fuzzy symmetric matrix of order n can always be diagonalised.

Rule 2: If any diagonalised matrix with their diagonal elements equal then the matrix is a scalar matrix.

Given : A fuzzy symmetric matrix A_F of order 2 has equal fuzzy eigen values.

By Rule 1 : A_F can always be diagonalised, let λ_1 and λ_2 be their fuzzy eigen values then

We get the diagonalized matrix = $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

Given $\lambda_1 = \lambda_2$

Therefore, we get = $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix}$

By Rule 2 : The given matrix is a scalar matrix.

Property 11:

The Eigenvector X of a matrix A_F is not unique.

Proof:

Let λ be the fuzzy eigen value of A_F, then the corresponding Eigenvector X such that $A_F X = \lambda X$.

Multiply both sides by non-zero scalar K,

$$K\,(A_F X) = K\,(\lambda X)$$

$$\Rightarrow A_F\,(KX) = \lambda\,(KX)$$

i.e., an Eigenvector is determined by a multiplicative scalar.

i.e., Eigenvector is not unique.

Property 12:

If $\lambda_1, \lambda_2, \ldots \lambda_n$ be distinct fuzzy eigen values of an $n \times n$ matrix then corresponding fuzzy eigen vectors $X_1, X_2, \ldots X_n$ form a linearly independent set.

Proof:

Let $\lambda_1, \lambda_2, \ldots \lambda_m \left(m \le n\right)$ be the distinct fuzzy eigen values of a fuzzy square matrix A_F of order n.

Let $X_1, X_2, \ldots X_m$ be their corresponding Eigenvectors we have to prove $\sum_{i=1}^{m} \alpha_i X_i = 0$ implies each $\alpha_i = 0, i = 1, 2, \ldots, m$

Multiplying $\sum_{i=1}^{m} \alpha_i X_i = 0$ by $\left(A_F - \lambda_1 I\right)$, we get

$$\left(A_F - \lambda_1 I\right) \alpha_1 X_1 = \alpha_1 (A_F X_1 - \lambda_1 X_1) = \alpha_1 (0) = 0$$

When $\sum_{i=1}^{m} \alpha_i X_i = 0$ is multiplied by

$$\left(A_F - \lambda_1 I\right)\left(A_F - \lambda_2 I\right)\ldots\left(A_F - \lambda_{i-1} I\right)\left(A_F - \lambda_{i+1} I\right)\ldots\left(A_F - \lambda_m I\right)$$

We get $\alpha_i \left(\lambda_i - \lambda_1\right)\left(\lambda_i - \lambda_2\right)\ldots\left(\lambda_i - \lambda_{i-1}\right)\left(\lambda_i - \lambda_{i+1}\right)\ldots\left(\lambda_i - \lambda_m\right) = 0$

Since λ 's are distinct, $\alpha_i = 0$

Since, i is arbitrary, each $\alpha_i = 0, i = 1, 2, \ldots, m$

$\sum_{i=1}^{m} \alpha_i X_i = 0$ implies each $\alpha_i = 0, i = 1, 2, \ldots, m$

Hence $X_1, X_2, \ldots X_m$ are linearly independent.

Property 13:

If two or more fuzzy eigen values are equal it may or may not be possible to get linearly independent Eigenvector corresponding to the equal roots.

Property 14:

Two Eigenvectors X_1 and X_2 are called orthogonal vectors if $X_1^T X_2 = 0$.

Property 15:

If A_F and B_F are $n \times n$ fuzzy matrices and B_F is a non singular fuzzy matrix, then A_F and $B_F^{-1} A_F B_F$ have same fuzzy eigen values.

Proof:

Characteristic polynomial of $B_F^{-1} A_F B_F$

$$= | B_F^{-1} A_F B_F - \lambda I | = | B_F^{-1} A_F B_F - B_F^{-1} (\lambda I) B_F |$$
$$= | B_F^{-1} (A_F - \lambda I) B_F | = | B_F^{-1} | \, | A_F - \lambda I | | B_F |$$
$$= | B_F^{-1} | \; | B_F | \, | A_F - \lambda I | = | B_F^{-1} B_F | \; | A_F - \lambda I |$$
$$= | I | \, | A_F - \lambda I | = | A_F - \lambda I |$$

= Characteristic polynomial of A_F

Hence A_F and $B_F^{-1} A_F B_F$ have same fuzzy eigen values.

3. CONCLUSION

In this paper, derived the properties of Eigen values and Eigen vectors for the fuzzy matrix, fuzzy matrix is vast area and the application of eigen values and eigen vectors of fuzzy matrix are Heat transfer equations, Control theory, vibration analysis, electric circuits, advanced dynamics and quantum mechanics, Moreover the eigen values of fuzzy matrix satisfies the properties of eigen values and eigen vectors is the main objective of this research paper.

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