Variants of CORDIC for Trigonometric Function Calculation and its Applications

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Abstract - Trigonometric functions have wide variety of applications in real life. Especially sine and copsine functions have been very useful in applications for medical science, signal processing, geology, electronic communication, thermal analysis, satellite communication and many more. There are certain applications where the power consumption of the hardware need to be reduced at the expense of accuracy. This paper presents a design that calculate sine and cosine values of a given angle using COordinate Rotation DIgital Computer (CORDIC) algorithm. This paper compares the variants of CORDIC which can be used for the calculation of the trigonometric functions of an angle such as bi-rotation CORDIC and four micro rotation CORDIC. It is found that these two methods increases the speed of computation at the expense accuracy.

Keywords-JPEG, CORDIC, SINE, COSINE,DCT,image compression,rotations

INTRODUCTION

Calculation of sine and cosine of given angle is an essential requirement in many areas of real life. In medical science, medical equipment that measures regular cyclical body functions like heartbeat, breathing etc. use sine and cosine waves. In signal processing, digital audio and high definition videos are based on sums of sine and cosine. Two popular methods of computing the sine and cosine of an angle involves the Taylor series method and look up table method, each have its own disadvantages like Taylor series involves the use of large number of multipliers and lookup table method increases the silicon area both increases the power consumption[12]. This paper presents hardware design for calculating sine and cosine value of given angle using CORDIC algorithm with limited hardware usage. As proposed by Volder[1] CORDIC is an iterative algorithm which is used to calculate trigonometric, exponential and hyperbolic function. Due to limited hardware architecture CORDIC requires low power as compared to their counterparts because for all calculation, CORDIC

architecture uses adder and shifter circuits. Since the modification in reference CORDIC provides only a

limited accuracy, it is suitable for application where a quantization is required[9]. The best example for this is JPEG image compression.

The paper has been organized as follows: section 2 introduces CORDIC algorithm, section 3 summarizes some of the previous work done in the same field. Section 4 explains variants of CORDIC and methodology Section 5 summarizes results of experiments and finally section 6 concludes the paper.

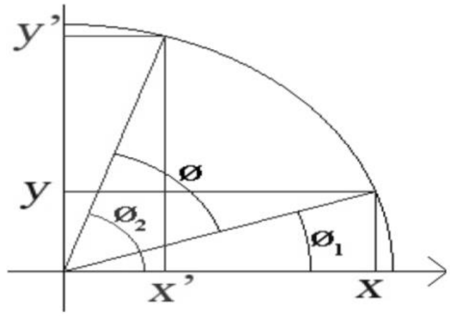
CORDIC algorithm

The CORDIC algorithm was proposed by Jack E volder [1] in 1959 for use in air borne system. As the time passed it have find its use in a wide variety of applications the basic CORDIC equations[1]

$$Y' = K(Y \cos\lambda + X \sin\lambda)$$

$X' = K(X \cos\lambda - X \sin\lambda)$

The only drawback of the CORDIC algorithm is the latency[1]



The initial value of x and y is (1,0) and through successive rotation the value we get sine and cosine of the angle which we give as input.

The analysis of equation in [9]

$(U_x)_{i+1} = (U_x)_i - \sigma.(U_y)_i .2^{-i}$ (1)

$(U_y)_{i+1} = (U_y)_i - \sigma.(U_x)_i .2^{-i}$ (2)

$\Phi_{i+1} = \Phi_i - \sigma \tan^{-1}(2^{-1})$ (3)

where $(U_x)_{i+1=}$Cos of the angle after n iterations,

$(U_y)_{i+1=}$Sin of the angle after n iterations.

i=1,2,3…………n;

REVIEWS

Jack E Volder[1] proposed the CORDIC algorithm, which has a latency problem. A.S.N Mokhtar (2013)[11] discussed how to increase the speed of the conventional CORDIC algorithm by combining third order approximation, In 2001, Kharrat et.al.[4]. proposed an optimized CORDIC implementation which offers reduction in silicon area and provides good precision of results but this approach was found suitable for up to 20 bits application. Pramod Kumar Meher, Sang Yoon Park (2013)[9] has suggested various variation in the reference CORDIC like four micro rotation, bi-rotation CORDIC (where the accuracy of the computation decreases but is sufficient for many application like image compression, audio compression etc). Pravin B. Pokle and N. G. Bawane (2012)[6] details about the JPEG (Joint Photographic Experts Group) process which is an lossy image compression technique which uses the DCT(Discrete Cosine Transform).

Varients of CORDIC and Methodology

Pramod Kumar Meher, Sang Yoon Park has suggested various variation in the reference CORDIC like four micro rotation, bi-rotation CORDIC.the sine and cosine angle using reference CORDIC typically involves 8 iterations to get a satisfactory result. This 8 iteration is the main reason for the latency in the CORDIC algorithm but it provides the best accuracy.

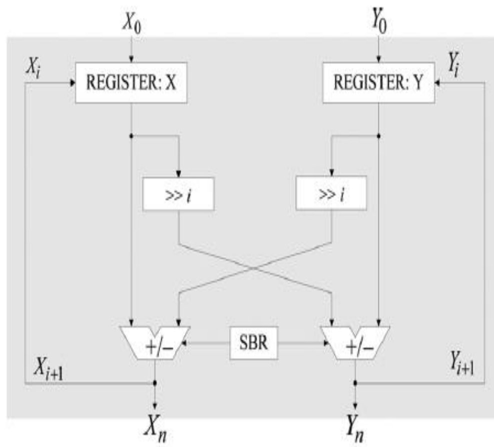


Fig1-Refernce CORDIC circuit

This paper also provides the tables for computation of the angles using 4-Micro rotations and bi- rotation this two algorithm increases the speed, but the accuracy is affected.

The equation for 4 micro-rotation is

$U_x)_{i+1} = (U_x)_i - (U_y)_i .2^{-k(i)}$ (4)

$(U_y)_{i+1} = (U_y)_i -.(U_x)_i .2^{-k(i)}$ (5)

The equation (3) can be eliminated as the table gives a s value which can be used instead of σ. In 4-micro only angles from 0-45 can be calculated the rest of the angle can be calculated using simple angle mapping described in [15].

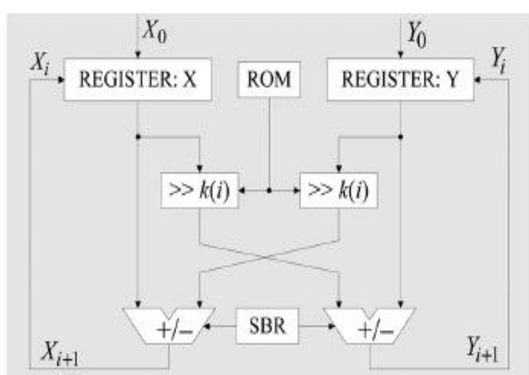


Fig 2-4-micro rotation

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The diagram of bi-rotation CORDIC is shown below:

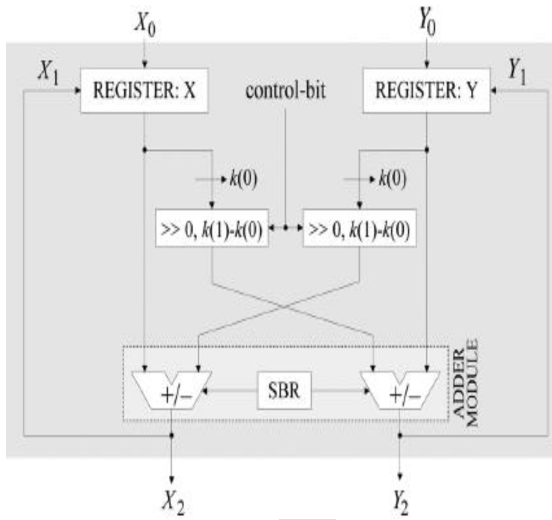

Fig 3. bi rotation CORDIC

In bi-rotation, out of the four values in the tables only two values is needed k(1), k(0). The input value for x_0 and y_0 (0.60725 and 0)[12] is given which is successively shifted by k(0), 0 and K(1)-k(0).and after 4 iterations we get the result in case of 4-micro rotation and after 2 iteration in case of bi-rotation and this can be effectively used for the computation of DCT in JPEG image compression.

RESULT OF SIMULATION

The following snapshots shows the result of simulation using the variants of CORIC multiplied by a multiplying factor (4096) as the FPGA's below SPARTAN 6 cannot process floating numbers directly. So the result shows the sine and cosine values of angle 37º

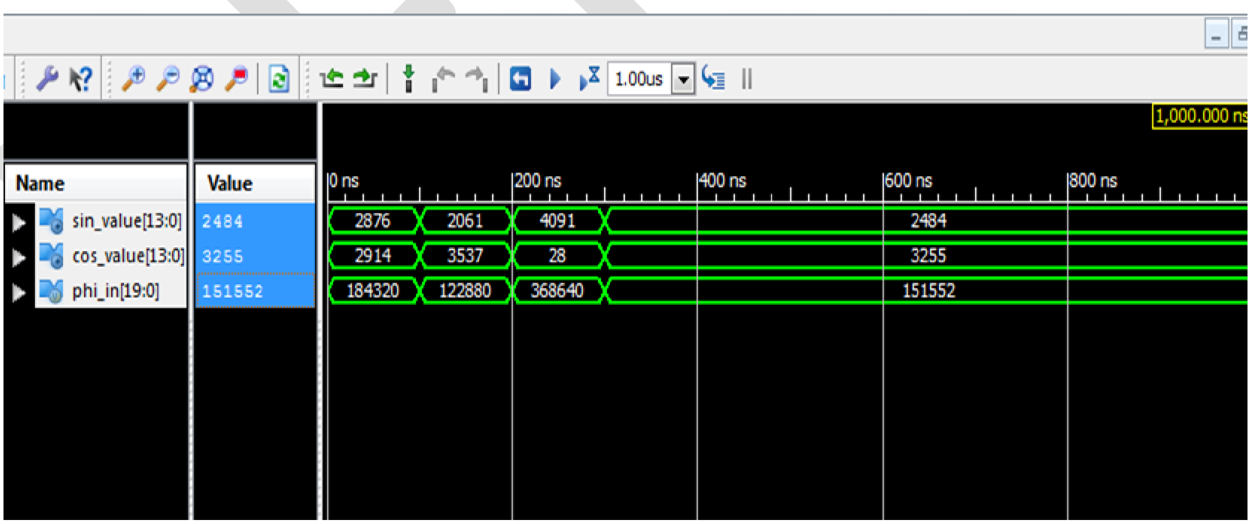

Fig 4. Simulation of reference CORDIC

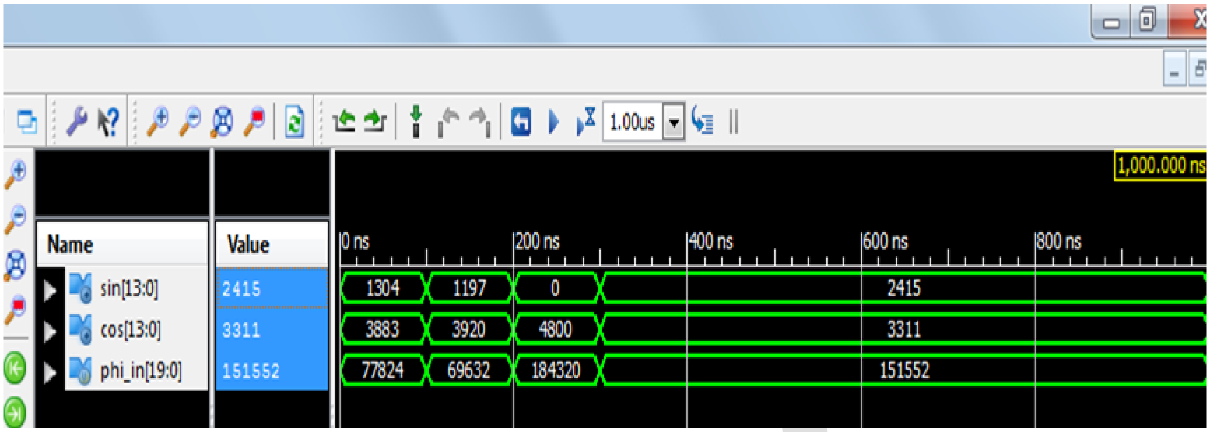

Fig 5. Simulation of 4-micro rotation CORDIC

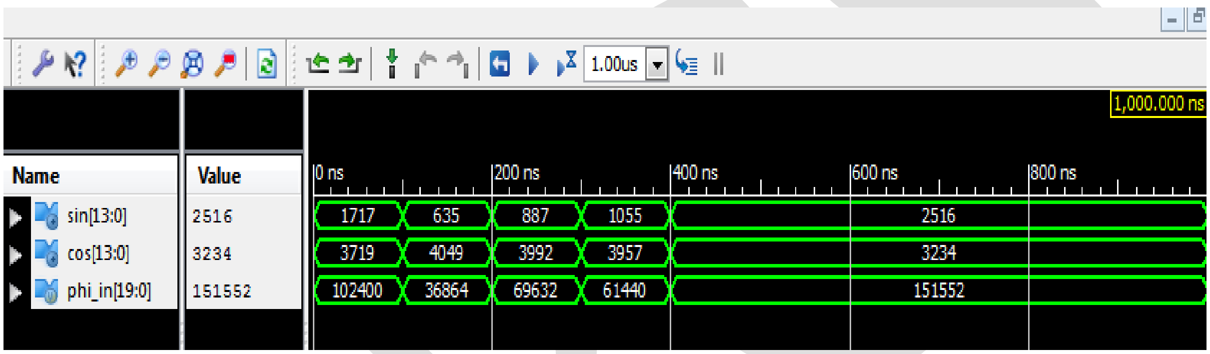

Fig 6. Bi-rotation CORDIC

Factors	**Variant of CORDIC**		
	Reference CORDIC	***4-micro rotation***	***Bi-rotation***
Number of slices used	1615	1054	175
Number of slices flip flops used	--	19	8
Number of 4 input LUT used	3170	1938	317
Number of bounded IOB used	48	48	48
Maximum combinational delay path (ns)	117.304	77.992	26.179

Table 1. Comparison of various CORDIC algorithms

The simulation result shows that the reference CORDIC gives the maximum accuracy at the expense of higher delay and utilizing 4 times the hardware as bi-rotation

CONCLUSION

From the simulation results, it can be concluded that the reference CORDIC provides the best accuracy for an angle but needs higher hardware and delay is also more,while the variants of the CORDIC reduces the hardware and delay but the accuracy will be reduced so these type can be effectively employed in application were a quantization is required like image compression or audio compression

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