Probabilistic Assessment on Flexural Strength of Steel Fiber Reinforced Concrete Members

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Abstract- This paper presents the reliability analysis of conventional reinforced concrete members and steel fiber reinforced concrete members subjected to flexural loading. First order Second Moment (FOSM) method is used to determine the reliability index. Monte Carlo simulation has been used to generate random samples of the variables in the present reliability analysis. C++ Computer programme has been written to obtain the Moment carrying capacity of a reinforced concrete beam of cross-sectional dimensions $250mm \times 500mm$ having 1% percentage of longitudinal reinforcement with 0.75% as the volume fraction of steel fibers. The cross sectional dimensions of the beam, compressive strength of the concrete, tensile strength of concrete are taken as variable parameters while keeping the other variables constant. The results revealed that there is an increase in cracking moment of approximately 17.5% after addition of steel fibres (volume fraction = 0.75 %) to the reinforced concrete beam and there is an increase of approximately 72% in the reliability index value, which suggests that steel fibre reinforced concrete beams are safer than reinforced concrete beams.

Key words: fibre reinforced concrete, reliability index, steel fibres.

1. Introduction

The evaluation of the safety of structures is a very importance task. It is one of the major concerns of engineers from a long time. The safety of a structure depends on the resistance, R of the structure and the action, S (load or load effect) on the structure. The action is a function of loads (live load, wind load etc.), which are random variables. Similarly, the resistance or response of the structure depends on the physical properties of materials, and the geometric properties of the structure which are also subjected to statistical variations, and are probabilistic in nature. Theory of probability provides a rational framework for accounting the uncertainties in loads and resistances. The uncertainty or incomplete information about the failure process is a result of complexity, imprecise measurements of the relevant physical constants and variables, and the indeterminate nature of certain future events.

The theory of structural reliability is widely used in many fields such as ship and offshore structures, architectural design, civil engineering, aerospace and mechanical engineering. In structural engineering, because of requiring long–term security commitments subject to various loads, the analysis of the reliability is particularly important. For a long time, the concept of reliability has been used to evaluate the quality of engineering structures. In the present study, the aim is to study the variations (randomness) of characteristic compressive strength, the breadth & depth of the beam, the load acting on the beam on the moment carrying capacity of reinforced concrete and fibrous reinforced concrete beam.

2. Literature review

Dmitri et al., (1997) reported that frames with longer external spans are less reliable in terms of progressive collapse resistance. The coefficient of variation of the structural resistance is approximately equal to or slightly less than that of the primary random variable, which is the steel strength in the case of the plastic failure mode and the concrete compressive strength in the case of the brittle one. Sofia et al., (1997) concluded that the amount of confining steel has negligible effect on the column reliability with respect to the ultimate strength, whereas the concrete compressive strength, the slenderness ratio, and the amount of longitudinal steel have a significant effect on the column reliability. The reliability of an HSC column is generally lower than that of the corresponding NSC column. Nataraja et al., (1999) reported that the addition of crimped steel fibers to concrete increased the toughness considerably and the increase in toughness was directly proportional to the reinforcing index. Monti and Santini (2002) proposed a possible methodology for the calibration of partial safety factors for the design of strengthening measures of reinforced concrete members using fibre-reinforced plastic (FRP). A first-order reliability method based optimization procedure was used to seek the solution of such a problem so that the target reliability is attained with the optimal FRP quantity. Wang et al., (2010) concluded that the resistance factor applied to the reinforced concrete remains unchanged, a separate resistance factor should be applied to the nominal strength of the CFRP plate to achieve the reliability index of approximately 3.0 (that is standard for reinforced concrete and steel beams designed for flexure).

3. Objective and Scope

The strength of reinforced concrete beam may vary due to variations in the material strengths of concrete and steel reinforcement, the cross sectional dimensions of concrete and steel, percentage of steel and cover to reinforcement. The effects of basic variables concrete strength, cross-sectional dimensions and loads are identified as significant.

The present investigation is aimed at predicting the reliability index of the reinforced concrete beam with varying compressive strength of concrete, cross-sectional dimensions and loads and to determine the reliability index for fibre reinforced concrete member under flexure and to compare it with that of reinforced concrete member under flexure. In the present study, FOSM has been used to evaluate Reliability Index (β). It is assumed that loads on the structure, the cross-sectional dimensions, compressive strength of concrete, cover to the beam follow normal distribution. The variation in the span of the beam and in the reinforcement provided is ignored. Statistical data of the variable parameters is presented in Table.1.

4. Methodology

4.1 Sampling Technique

The reliability indices of a system can be calculated using two basic approaches, direct analytical techniques or stochastic simulation. The difference between the analytical and simulation approaches is the way in which the reliability indices are evaluated. Analytical techniques represent the system by a mathematical model, which is often simplified, and evaluate the reliability indices from this model using direct mathematical solutions.

Simulation techniques, on the other hand, estimate the reliability indices by simulating the actual process and random behaviour of the system. The method therefore treats the problem as a series of real experiments conducted in simulated time. It estimates probability and other indices by counting the number of times an event occurs. Monte Carlo simulation has been used to generate random samples of the variables in the present reliability analysis of the R.C beams

4.2 Reliability Study of RC Beams

4.2.1 Assumptions

In the reliability analysis and design of structures, it is important to first study the distribution pattern of the components such as loads and resistance. The strength of a structural member will vary from the calculated value or nominal strength due to variations in material strengths and in the dimensions of the members, as well as variables inherent in the equations used to calculate the strengths of the members. The compressive strength of concrete, breadth of the beam, width, Depth and cover of the beam along with the loads acting on the beam are all considered to be normally distributed.

4.2.2 Design of Beam

The beam was designed as per IS 456:2000 code provisions. The beam was checked for safety in deflection and crack width.

4.2.2.1 Control of Cracking

Cracking is a very complex phenomenon. The practical objective of calculating crack width is merely to give guidance to the designer in making appropriate structural arrangements and in avoiding gross errors in design, which might result in concentration and excessive width of flexural crack. The formula can be used provided that the strain in the tension reinforcement does not exceed $0.8\,\sigma_y/E_s$.

Design surface crack width, W_{cr}

$$W_{cr} = \frac{3a_{cr}\varepsilon_m}{1 + \frac{2(a_{cr} - C_{min})}{h - x}}$$

Where, a_{cr} = distance from the point considered to the surface of the nearest longitudinal bar

C_{min} = Minimum cover to the longitudinal bar,

ε_m = Average steel strain at the level where cracking is being considered,

$$\varepsilon_m = \varepsilon_1 - \frac{b(h-x)(a-x)}{3E_sA_s(d-x)}$$

h = overall depth of the member, and

x = depth of neutral axis.

A_s = Area of tension reinforcement,

b = width of the section at centroid of the tension steel,

ε_1 = Strain at the level considered, calculated ignoring the stiffening of the concrete in the tension zone,

a = distance from the compression face to the point at which crack width is being calculated,

d = effective depth.

Crack width has been calculated at three points:

(a) Directly under a bar on tension face of concrete

(b) At a point on the tension face midway between the two bars

(c) At the bottom corner.

In general, the surface crack width should not exceed 0.3 mm. For very aggressive environment, the assessed surface width of cracks at points nearest to the main reinforcement should not exceed 0.004 times the nominal cover to the main reinforcement.

4.2.2.2 Control of Deflection

The permissible deflection is governed by the amount that can be tolerated by the interacting components of the structure. It is essential to consider both the short-term and long-term deflections. The deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. The deflection shall generally be limited to the following:

(a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the, supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

(b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span/350 or 20 mm whichever is less.

4.3 Reliability Study of SFRC Beam

4.3.1 Assumptions

There are few assumptions which have been made in the design of steel fiber reinforced concrete beam. The aspect ratio of fiber has been assumed as 90. The volume fraction of steel fibers has been assumed as 0.75%.

The equivalent modulus of elasticity, E_{eq} is estimated using the formula used by Teng (2004)

$$E_{eq} = E_f V_f + (1 - V_f) E_m$$

Where, V_f = volume fraction of the fiber,

E_f = Modulus of Elasticity of the fiber used,

E_m = Modulus of Elasticity of the matrix (Concrete)

The residual tensile strength of fiber reinforced concrete has been calculated by the formula suggested by Naaman (2003) which is

$$f_r = \lambda_1 * \lambda_2 * \lambda_3 * \tau * F$$

Where f_r = residual tensile strength of fibrous concrete

λ_1 = Expected pullout length ratio = 0.25

λ_2 = Efficiency factor of orientation in the cracked state = 1.2

λ_3 = Group reduction factor associated with the number of fibers pulling out per unit area = 1

τ = average bond stress of a single fiber embedded in the concrete.

The above values of $\lambda_1,\ \lambda_2\ and\ \lambda_3$ are applicable upto an aspect ratio of 100.

The compressive strength of steel fiber reinforced concrete has been found obtained by the formula suggested by Nataraja et al., (1999) which is

$$f_{c'} = f_c + (6.913 * F)$$

Where $f_{c'}$ = compressive strength of fiber reinforced concrete,

f_c = Compressive strength of plain concrete,

F = Fiber factor = $V_f * aspect\ ratio.$

This formula for calculating compressive strength of fiber reinforced concrete is valid for concrete strength upto 50MPa. The beam was designed considering the above assumptions and it was checked deflection and cracking criteria

5. Results and Discussion

A C++ Computer programme has been written to obtain the Moment carrying capacity of a reinforced concrete beam of cross-sectional dimensions $250mm \times 500mm$ for 1% of longitudinal reinforcement. For steel fibre reinforced concrete the volume fraction of steel fibers with 0.75% and an aspect ratio of 90 are used. The cross sectional dimensions of the beam were given as inputs to study the variation in the moment carrying capacity of the beam keeping the other variables constant.

The mean and standard deviation of the different parameters considered in this are presented in table. 1. The limiting moment has been obtained from the equation, $M_{lim} = 0.138 * f_{ck} * b * d^2$, considering $f_{ck},\ b,\ d$ as random variable and the result obtained for RC beam and SFRC beam respectively are presented in figure 1 a, b. The maximum bending moment carrying of the beam has been obtained by the equation, $M_u = 1.5 * 0.125 * w * L^2$, considering load acting on the beam, w as random variable. The variation in the cracking moment observed for RC beam and SFRC beam respectively is presented in figure 2 a, b.

Comparison of Frequency Distribution for Bending Moment (Action) and Limiting Moment (Resistance) of the Beam for RC beam and SFRC beam respectively is presented in figure 3 a, b. It is to be noted that the shaded portion in the plot represent the failure region.

Comparison of Results

From the analytical study of RC and SFRC beams the reliability index is calculated using the resistance and action of the beams. The comparison of the same is presented in the table 2.

6. Conclusions

The investigation tries to compare the reliability (safety) index – β for a normal reinforced concrete beam with that of steel fiber reinforced concrete beam. The following conclusions have been drawn based on the observations and the results of the study.

The cracking moment for reinforced concrete beam in obtained as 32.5kN.m whereas the same for a steel-fiber reinforced concrete beam is 38.1 kN.m. Thus, there is an increase in cracking moment of approximately 17.5% with addition of steel fibers (volume fraction = 0.75 %) to the reinforced concrete beam. The increase in cracking moment is due to the fibers distributed in the matrix which absorb a large amount of energy for crack formation as well as crack propagation. This aspect of steel fibers in concrete increases the serviceability of the structure.

The reliability index has been calculated by First Order Second Moment method and it is found to be 1.0578 for reinforced concrete beam whereas the same for a steel-fiber reinforced concrete beam is 1.75162. Hence, there is an increase of approximately 72% in the reliability index value, which suggests that steel fiber reinforced concrete beams are safer than reinforced concrete beams or for the same reliability the steel fiber reinforced concrete beam can be designed with economical dimensions.

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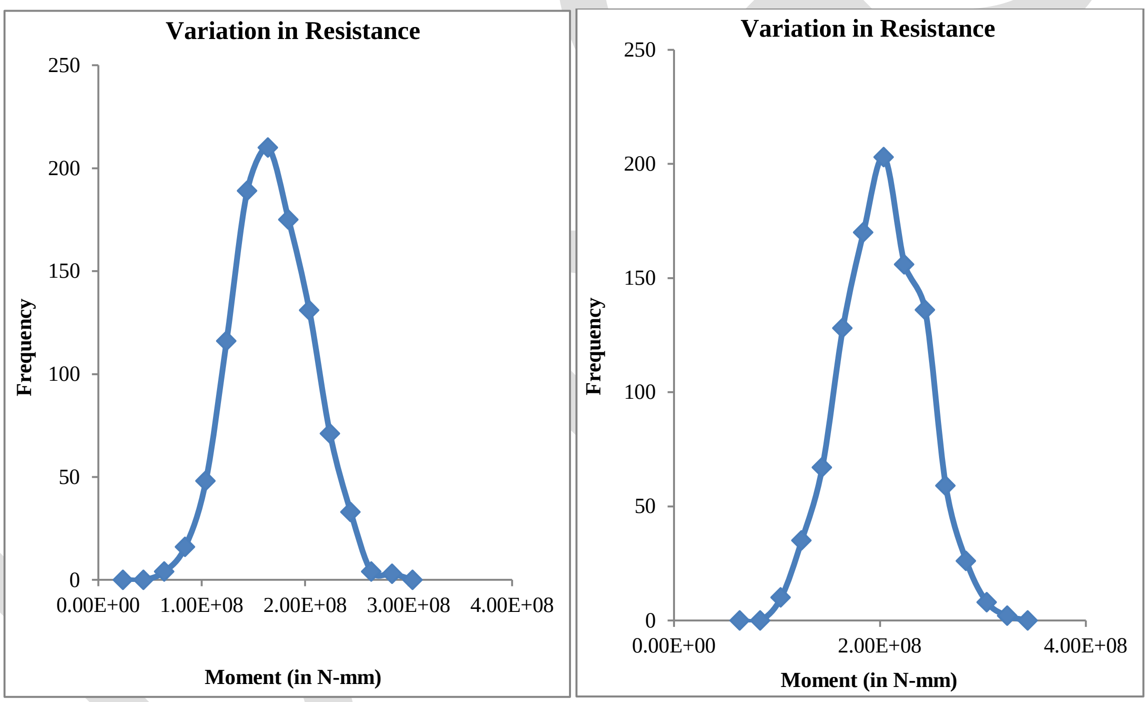
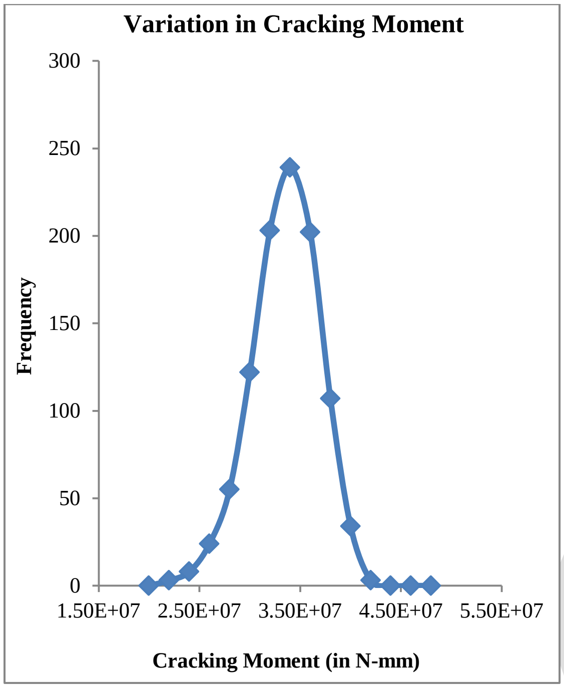
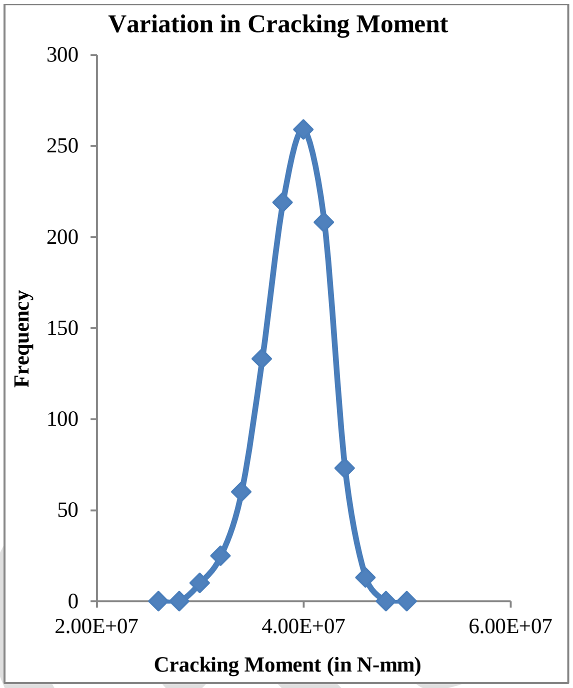


A) RCC Beam **b) SFRC Beam**

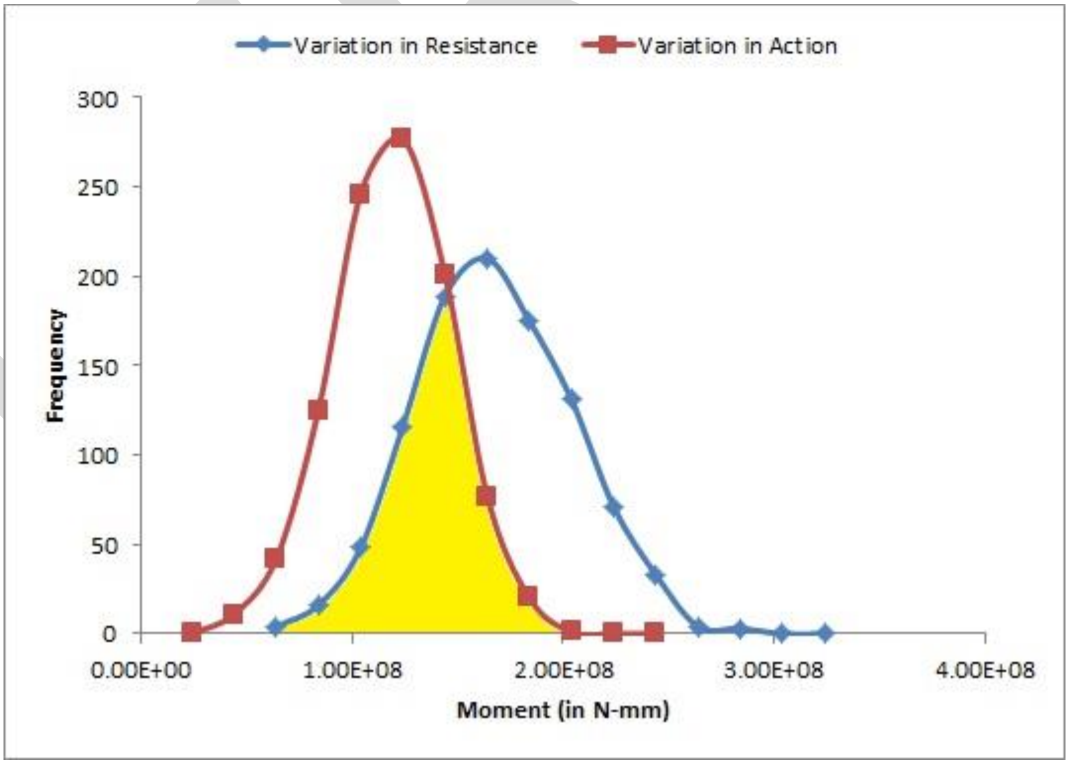
Figure 1: Normal distribution curve for limiting moment of the beam





a) RCC Beam **b) SFRC Beam**

Figure 2: Normal distribution curve for cracking moment of the beam



a) RCC Baem

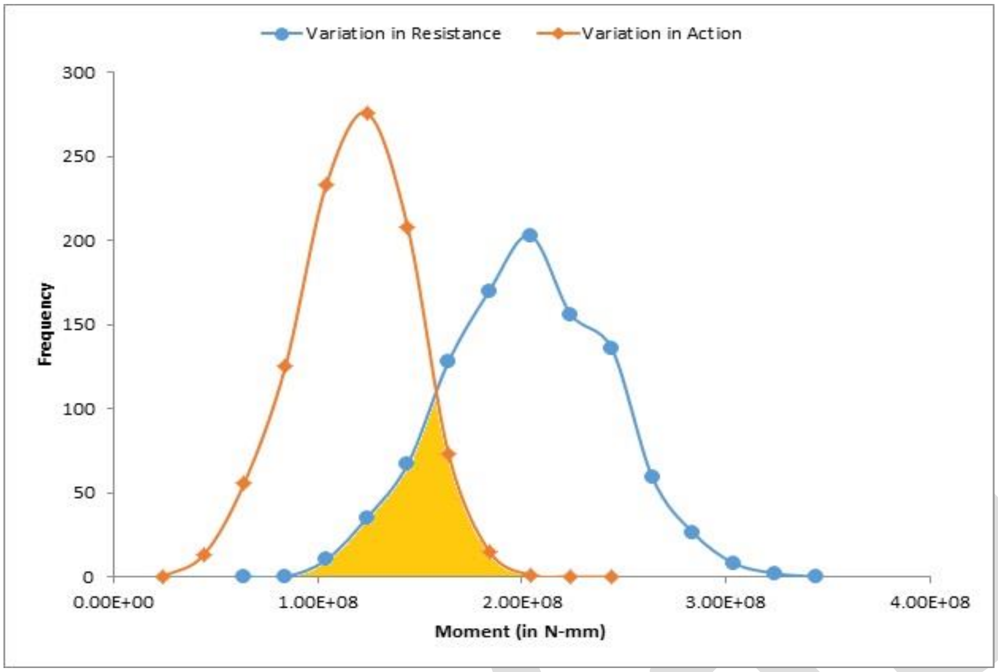


Figure 3: Normal distribution curve for Bending Moment (Action) and Limiting Moment (Resistance) of the Beam

Table.1- Statistical data of the variable parameters (ref. Ranganathan (1990)

b) SFRC Beam

Variable parameter	Mean	Standard Deviation	Probability Distribution
Depth (mm)	450	9.38	Normal
Breadth (mm)	250	9.47	Normal
Cover to longitudinal Reinforcement	25	8.41	Normal
Compressive strength of concrete (MPa)	20	4.1034	Normal
Loading (kN/m)	20	6	Normal

Table 2 - Comparison of the results of the RC and SFRC beams

	RC Beam	SFRC Beam
Average of Resistance, μ_R (N.mm)	157833230.9	192918219.9
Average of Action, μ_S (N.mm)	109037313.5	108089948.1
Average of Margin of Safety,$\mu_M = \mu_R - \mu_S$ (N.mm)	48795917.4	84828271.8
Standard Deviation of Resistance,σ_R (N.mm)	37244254.44	39537503.07
Standard Deviation of Action,σ_S (N.mm)	27219570.52	27965773.06
Standard Deviation for Margin of Safety,$\sigma_M = (\sigma_R^2 + \sigma_S^2)^{1/2}$(N.mm)	46130678.6	48428283.18
Reliability Index, $\beta = \mu_M / \sigma_M$	1.0578	1.7516
Average of Cracking Moment (N.mm)	32518126.3	38161033.3
Standard Deviation of Cracking Moment (N.mm)	3302483.341	3059554.652