International Journal of Engineering Research and General Science Volume 3, Issue 1, January-February, 2015
ISSN 2091-2730

PECULIARITY OF NUCLEUS IN ALTERNATIVE RINGS

Y.s.n.satyanarayana #[1] Dr.A.Anjaneyulu[#2] Dr.D.prabhakaraReddy[#3]

[#1]Assistant professor in mathematics, Brindavan institute of Technology & science, Kurnool, Andhra Pradesh, INDIA.yeemanisatyam@gmail.com,9848560502.

[#2]Department of Mathematics, VSR &NVR college,Tenali, Andhra Pradesh, INDIA.

#3 Lecture in mathematics, IT Department, Nizwa college of technology, Nizwa, OMAN

ABSTRACT: In this paper we have proved that the peculiar property of nucleus (N) in an alternative ring (R) i.e nucleus contracts to centre C when alternative ring is octonion and nucleus expands to whole algebra when the alternative ring is associative.

KEYWORDS:, commutator, c Associator entre, Nucleus, Octonion, Moufang laws, Alternative ring.

INTRODUCTION: Alternative rings arose out of the work of Ruth Moufang in 1930's When he put his attention mainly on multiplicative structure of alternative division ring .The next prime mover was E.G. Goodaire[1]He shows that alternative loop algebras does not exists. E.Kleinfled[2,3,4,5], R.H. Bruck[6],Emil Artin and Macdonold gives a beautiful characterization of alternative algebras. Armin Thedy[7,8,9] discussed about right alternative rings.Slater[10,11] gives an idea about purely alternative rings concept.Kevin Mc Crimmon [12,13]in his A taste of Jordan algebras mention peculiarities of alternative nucleus.Alternative rings derive their name from the fact that in an alternative ring the associator is an alternating function through linearization.

PRELIMINARIES: A ring R is said to alternative ring in which left and right alternative laws satisfies.

$$(p^2q) = p(pq)$$

$$(qp^2) = (qp)p$$

In nonassociative ring theory two important functions are commutator and associator defined as

$$(pq) = pq - qp$$

$(pqr) = (pq)r - p(qr)$ for all p,q,r in R.

An alternative algebra is automatically flexible which satisfies the commutation relation $(pq)p = p(qp)$ Alternatively is equivalent to the associator (p,q,r) being an alternating multilinear function of its arguments i. e in the sense that it vanishes if any two of its variables are equal

$$(p,p,q) = (q,p,p) = (p,q,p) = 0$$

The very interesting and peculiar property is that in non associative ring the centre and ,nucleus are associative defined as follows

Nucleus(N) = { $\alpha \in R / (\alpha,\beta,\beta) = (\beta,\alpha,\beta) = (\beta,\beta,\alpha) = 0$ }

Centre (c) = $= \{c \in N / (C,R) = (C,R,R,) = 0$ }

Nuclearity conditions can be written in terms of associators as $(n,p,q)=(p,n,q)=(p,q,n)=0$ so alternation nuclearity $(p,q,n)=0$ reduces to $(n,p,q)=0$ in alternative algebra. In general these identities are quadratic in p so they automatically linearize and all scalar extensions of alternative algebras are again alternative

The fundamental and most extremely useful tool in nonassociative algebra is linearization using this concept we replace a repeated variable or an identity by the sum of two variables in order to obtain another identity like $(p,r,q)=-(r,p,q)$,

$$(p,q,r)=-(p,r,q)$$

Alternative rings automatically satisfy flexible identity known as Moufang lemma.

$$(p,q,p)=0$$

$$\Rightarrow \quad (p,q,p)=-(p,p,q)=0$$

The flexible identity says $pq.p=p.qp$ in other words the expression pqp is unambiguous in an alternative ring.

Kleinfield function f : $R^4 \rightarrow R$ defined as

$$f(\omega,p,q,r)=(\omega p,q,r)-p(\omega,q,r)-(p,q,r)\omega ---------------(1)$$

$$(p^2,q,p)=p(p,q,r)+(p,q,r)p ------------(2)$$

If the characteristic is not two, the liberalized relation implies the flexible property valid in any ring known as Teichmuller identity

$$(\omega p,q,r)-(\omega,pq,r)+(\omega,p,qr)=\omega(p,q,r)+(\omega,p,q)r ----------(3)$$

Alternative algebra satisfy the Moufang left, middle, right laws and bumping formula which plays a vital role they are

$$((pq)p)r-p(q(pr)) ----------------(4)$$

$$(pq)(rp)=p(qr)p -----------------(5)$$

$$((pq)r)q-p(q(rq) -------------------(6)$$

$$(p,q,rp)=p(q,r,p) ----------------(7)$$

The respective proofs (3) to (7) are as follows,

$$(\omega p,q,r)-(\omega,pq,r)+(\omega,p,qr)$$
$$((\omega p)q)r-(\omega p)(qr)-((\omega(pq))r-\omega((pq)r))+((\omega p)(qr)-\omega(p(qr)$$

$$((\omega p)q)r-(\omega p)(qr)-(\omega(pq))r+\omega((pq)r))+(\omega p)(qr)-\omega(p(qr))$$

$$((\omega p)q)r-(\omega(pq))r+\omega((pq)r))-\omega(p(qr))$$

ISSN 2091-2730

$$((\omega p)q-\omega(pq))r+\omega((pq)r-p(qr)$$

$$(\omega,p,q)r+\omega(p,q,r)$$

Moufang laws

$$((pq)p)r-p(q(pr))=((pq)p)r-(pq)(pr)+(pq)(pr)-p(q(pr))$$

$$=(pq,p,r)+(p,q,pr)$$

$$=(q,p,r)p+(pr,p,q)$$

$$=(q,p,r)p+(r,p,q)p=0$$

Similarly middle and right laws,

$$(pq)(rp)-p(qr)p=(pq)(rp)-((pq)r)p+((pq)r)p-p(qr)p$$

$$= -(pq,r,p)+(p,q,r)p$$

$$= (pq,p,r)+(p,q,r)p$$

$$=(q,p,r)p+(p,q,r)p=0$$

$$((pq)r)q-p(q(rq)=((pq)r)q-(pq)(rq)+(pq)(rq)-p(q(rq))$$

$$=(pq,r,q)+(p,q,rq)$$

$$=-q(p,q,r)-q(r,q,p)=0$$

Bumping formula as $(p,q,rp)=-(p,q,pr)-(r,q,p^2)$

Using Jordan algebra and linearization

$$=(p,pr,q)-(p^2,r,q)$$

By alternativity and left alternativity,

$$=(p^{2^z}r)q-p((pr)q)-(p^2r)q+p^2(rq)$$

$$=-p((pr)q)+p(p(rq))$$

$$=-p((pr)p-p(rq))$$

$$=-p(p,r,q)$$

By using alternativity

$= p(q,r,p) = x(y,z,x).$

MAIN RESULTS:

Lemma: If R *is* an alternative with nucleus $N = Nuc(R)$ then for any element $n \in N,\ p,q,r \in R.$, it satisfies

(1) Nuclear slipping formula

$$n(p,q,r) = (np,q,r) = (pn,q,r)$$

(ii) The nucleus is commutator closed $(N,R) \subseteq N$

(iii) It satisfy nuclear product relations $(N,X)p \subseteq N, (p,n)(p,q,r) = 0$ for any $p,q,r \in R.$.

(iv) Nuclear commutators absorb R and annihilate associators.

$$(N,N)R \subseteq N, (N,N)(R,R,R) = 0$$

(v) If C satisfy $(C,R) \subseteq C \subseteq N$ then If $c \in (C,C)$ it has $c^{-1} \in R$ then $R = C$ is associative.

PROOFS: From the equation (3)

$$(\omega p,q,r) - (\omega, pq,r) + (\omega, p,qr) = \omega(p,q,r) + (\omega, p,q)r$$

+

Put $\omega = n \in N_\alpha$

$$(np,q,r) - (n, pq,r) + (n, p,qr) = n(p,q,r) + (n, p,q)r$$

$$\Rightarrow (np,q,r) = n(p,q,r)$$

$$\Rightarrow (np - pn,q,r) = n(p,q,r)$$

$$\Rightarrow (np,q,r) - (pn,q,r) = n(p,q,r)$$

$$\Rightarrow (np,q,r) = (pn,q,r) = n(p,q,r)$$

Known as nuclear slipping formula.

ii) $(N,R) \subseteq N$

since $(n,p)p \in N \Leftrightarrow ((n,p)p,R,R) = 0$

$$\Leftrightarrow (n,p)(p,q,r)$$

By using nuclear slipping formula

Hence $(n,p) \subseteq N$

$$\Rightarrow (N,R) \subseteq N$$

(iii) From equation (3) and (1)

$$f(p,q,r,n) = (pq,r,n) - q(p,r,n) - (q,r,n)p = 0$$

$$\Rightarrow f(n,p,q,r) = (np,q,r) - p(n,q,r) - (p,q,r)n = 0$$

which gives $(np,q,r) = (p,q,r)n$

Similarly $(pn,q,r) = n(p,q,r)$

$$(pn,q,r) - (p,nq,r) + (p,n,qr) - p(n,q,r) - (x,n,q)r = 0$$

$$\Rightarrow (pn,q,r) - (p,nq,r) = 0$$

So $(pn,q,r) = (p,nq,r)$,

since $(pn,q,r) = (nq,r,p) = (q,r,p)n = (p,q,r)n = (np,q,r)$

Thus we obtain $((n,p)q,r) = 0$ thus $(p,n) \in N$

Now by using equation (2)

$$((qp,n) = q(p,n) + (q,n)p$$

$$\Rightarrow ((q,p,n),p,r) = 0$$

$$\Rightarrow ((q(p,n)p,r) + ((q,n)p,p,r) = 0$$

$$\Rightarrow (p,n)(q,p,r) + (p,p,r)(q,n) = 0$$

$$\Rightarrow (p,n)(q,p,r) = 0$$

(iV) Linear zing $p \to p + m$ for $m \in N$ in result (2) gives (7)

$$(N,p)p \subseteq N \ ,(N,p+m)(p+m) \subseteq N\ , \ ((N,p)+(N,m))(p+m)$$

$$(N,p)p + (N,p)m + (N,m)p + (N,m)m$$

$$\Rightarrow (N,p)m + (N,m)p = 0$$

$$\Rightarrow (N,p)m = -(N,m)p \subseteq N$$

Similarly, we get

$$(n,m)(p,q,r) = -(n,p)(m,q,r)$$

$\Rightarrow (n,m)(p,q,r) = 0$ Since $(N,N)R \subseteq N$

$$\Rightarrow (n,m)(R,R,R) = 0$$

(V) If $C \supseteq cR \supseteq c(c^{-2}(cR))$

$= (cc^{-2}c)(R)$

Using left Moufang

$= IR = R$

$\Rightarrow N \supseteq C \supseteq R \supseteq N$

$\Rightarrow N = C = R.$

Thus all these properties are also true in associative algebra where $(p,q,r) = 0$ and

$N = R$ and in octonion algebra where $N = cen(R), (N,R) = 0.$

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