

A STRUCTURAL WEIGHT BASED MINIMUM DISTANCE PATH METHOD USED FOR IDENTIFICATION, ISOMORPHISM AND DISTINCT MECHANISMS OF KINEMATIC CHAINS

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Abstract— This work presents a new method to identify distinct mechanism(DM) desired as in from the given family of kinematic chains(KCs). In this method the given KCs are represented in the form of structural link valued shortest path distance matrix (SLVSPDM) thus representing the identification number of each link. The equivalent links have the same identification number resulting a distinct mechanism. Same identification number of KCs resulting isomorphism . The method is examined for one degree of freedom (1-D.O.F), 8 links,16 Kinematic planner kinematic chains. Result is useful for designer to deduct isomorphism and distinct mechanism from the available kinematic chains family.

Keywords— Kinematic chain, Isomorphism, Distinct Mechanism, Structural Weighted Link Values.

INTRODUCTION

Structural synthesis of kinematic chain(KCs) and mechanism has been the subject of a number of studies in recent years. One important aspect of structural synthesis is to develop all possible mechanism derived from a given kinematic chain so that the designer has the liberty to select the best or optimum mechanism depending upon the application. In the course of development of mechanisms, duplication or isomorphism among kinematic chains with same number of links is necessary to prevent duplication and omission of a chain which is mechanically more useful than its isomer. A lot of literature related to isomorphism detection and detection of distinct mechanism (DM) is available but still there is scope for an efficient simple and reliable method and this paper is an attempt in this direction. Graph theory (Hsu and Lam, 1992)¹¹ has been widely adopted for representation of mechanism. Therefore, plenty of literature dealing with this topic in the area of graph theory as well as kinematic exists. The characteristic polynomial methods were developed by Uicker and Raicu(1975)⁽²²⁾, Yan and Hall(1981,1982)^(23,24), Mruthyunjaya (1984a, 1984b)^(13,14) and Mruthyunjaya and Balasubramaniam (1987)⁽¹⁶⁾. These methods have the disadvantage of dealing with large numerical and later counter examples were also reported by (He et al.(2005)⁽⁸⁾). Canonical code approaches (Ambeker and Agrawal, 1987)⁽¹¹⁾ require highly sophisticated algorithms and greater computational effort, when large applied to large kinematic chain. Hamming number technique (Rao and Rao 1993a, 1993b)^(19,20) is very reliable and computationally efficient, however when the primary Hamming string fails, the cumbersome computation of the secondary Hamming string is needed. Hsu (1993a, 1993b)^(9,10) using the concept of admissible graphs synthesized all possible graphs of planetary gear trains by the process of edge transformation. Then, the structural codes of graphs are used to identify the isomorphism. The adjacent-chain table method (Chu and Cao, 1994)⁽³⁾ had been proposed to identify isomorphism, but it is not suitable for computerised structural synthesis. The fuzzy logic method (Rao, 2000)⁽¹⁸⁾ requires computation of not only the first adjacency matrix but also the adjacency matrix of higher, even up to N/2. The method of eigenvectors and eigenvalues of adjacency matrices (Chang et al., 2002; Cubillo and Wan, 2005)^(2,4) possesses the advantages of using standard matrix theory, but it does not belong to the code based method and it is hard to analysis the topological structure of kinematic chains. The unconventional approaches, such as the artificial neural networking approach (Kong et al., 1999)⁽¹²⁾ and the genetic algorithm (Rao, 2000)⁽¹⁷⁾ are also applied to isomorphism identification, but the effectiveness of these methods still needs testing (Mruthyunjaya, 2003)⁽¹⁵⁾. Sunkari and Schmidt (2006)⁽²¹⁾ first time established the reliability of the existing spectral techniques for the isomorphism detection. Ding and Hung (2009)⁽⁶⁾ addresses the problem of isomorphism identification by finding a unique representation of graph. The unique representation of graph database feasible. It remains efficient even when the links of kinematic chains increases into the thirties. Hasan and Khan (2009)⁽⁷⁾ presented a method based of degrees of freedom of kinematic pairs, Darger et al. (2010)⁽⁵⁾ proposed a method based on first and second adjacency value of links, but no mathematical proof was presented. In this paper a new topological description (based on minimum path distance link structural value matrix) (MPDLSVM) is proposed. The proposed method has

following very fruitful characteristics:

2. Definition of Terminology:-

The following definition are to be understood clearly before applying this method. Various definitions with their abbreviations are given below.

1. Degree of link (D): A numerical value for the link, based on its connectivity to other links. Therefore quaternary link has degree equal to four and ternary link has equal to three.
2. Link value (LV) - For a particular link it is defined as the ratio of number of design parameter associated with the link to be degree of that link. The link values of various types of links are determined by equation 1 and listed in Table-1.

Table-1

Type of link	Link value
Binary link	$1 \div 2 = 0.5$
Ternary link	$3 \div 3 = 1$
Quaternary link	$(4+1) \div 4 = 1.25$
Quinternary link	$(5+2) \div 5 = 1.4$

3. **Link Structural value (LSV)** - For a particular link it is defined as the product of a link value to degree of links are connected to link in question. It can be explained by the example 1(Watt mechanism) shown in Fig.1.

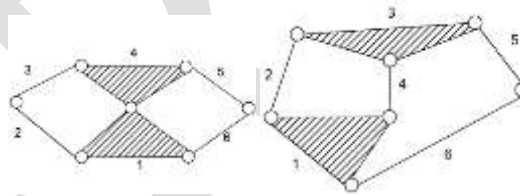


Figure-1 Watt Mechanism

Figure-2 Stephen chain

Link 1 of figure 1 is connected with three links, one ternary and two binary hence connected degree of links to link 1 is

$$(3+2+2) = 7 \text{ -----(1)}$$

And link structural value of link 1 is equal to

Link value \times connected degree of links to link in question

For link no 1 of watt mechanism shown in figure1 = 1(from table 1) \times 7(from eq.1) = 7

$$\text{-----(2)}$$

Similarly for link 2,3----6 are

$$0.5 \times (3+2) = 2.5, 0.5 \times (3+2) = 2.5, 1 \times (3+2+2) = 7, 0.5 \times (3+2) = 2.5 \text{ and } 0.5 \times (3+2) = 2.5.$$

$$\text{-----(3)}$$

4.- Structural weighted link label V_i – Usually the canonical labels depends only on the connectivity of the links being labeled together with its immediate neighbor. However, in a closed kinematic chain links are connected by joints, so as form loop and every link has a distinct relation with every other link in the form of minimum distance between them which is constant and is presented to have a matrix is called link path matrix of the chain. Considering this in mind the usual canonical labeling is extended to include all links of the chain. Canonical label V of a link of kinematic chain, is defined to have a sum of path weighted with structural link value. Each link L ia assigned a label V_i as follows-

$$V_i = \sum_{j=1}^n (LSV)_j \times D_{ij}$$

-----(4)

Where (LSV) is the link structural value already explained in section 3 and by equation 2.

And D is the minimum path from link to link.

Hence Kinematic chain structural weighted labeling is

$$KCSWL = \sum (V_i)^2$$

-----(5)

For example for a graph of Watt mechanism shown in Fig.1

LSV of Watt mechanism is shown in Table 2 by equation 2-

Table-2

Link No.	1	2	3	4	5	6
SLV	7	2.5	2.5	7	2.5	2.5

-----(6)

Minimum distance matrix of watt mechanism shown in fig,1 is-

Link →	1	2	3	4	5	6
↓						
1	0	1	2	1	2	1
2	1	0	1	2	3	2
3	2	1	0	1	2	3
4	1	2	1	0	1	2
5	2	3	2	1	0	1
6	1	2	3	2	1	0

-----(7)

Hence Structural weighted link labeled matrix by equation no.4,6 and 7

Link →	1	2	3	4	5	6	V _i
↓							
1	0	2.5	5	7	5	2.5	22
2	7	0	2.5	14	7.5	5	36
3	14	2.5	0	7	5	7.5	36
4	7	5	2.5	0	2.5	5	22
5	14	7.5	5	7	0	2.5	36
6	7	5	7.5	14	2.5	0	36

-----(8)

And KCSWL (by equation 5) is- 6132

Hence identification code of watt mechanism is

6132/4(36)/2(22)/ and distinct mechanisms are 2.

Consider Stephenson chain shown in Fig.2

By using equations 4,6,7 and 8

Identification code is 4981/2(32.5)/2(28)/2(25.5)/

And KCSWL is 4981

5.Detection of Isomorphism:- The set of link labels (V_i) can directly be used to distinguish kinematic chains. To make it more meaningful the structural weighted link labels calculated above can be combined to generate a numerical code for a kinematic chain. Squared sum of link(V_i) defined as the kinematic chain structural weighted label (KCSWL) as an index for testing of isomorphism. If two chains having identical KCSWL will be isomorphic to each other.

6. Application of concept is illustrated with no. of several examples:-

By comparison of KCSWL of Watt chain and Stephenson chain it is clear from the result that two chains are non-isomorphic. And Watt chain having two distinct mechanisms (by inspection of structural weighted link labels- /4(36)/2(22)/), Stephenson chain having three distinct mechanisms (SWLL- /2(32.5)/2(28)/2(25.5)/).

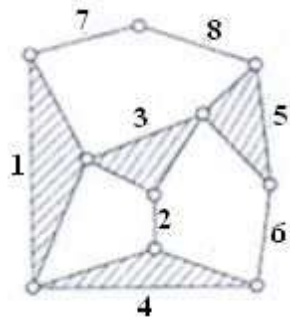


Fig.3(a)

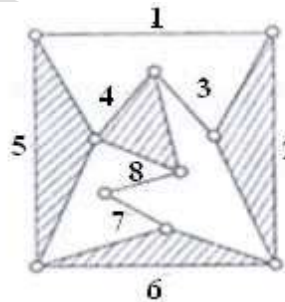


Fig.3(b)

Eight Link Isomorphic chains

For Fig. 3(a) the structural weighted link labels or labels are-

[77.5/72.5/66/64.5/56.5/54.5/48.5/48/]

KCWL is 30597.5 and identification code is

488[77.5/72.5/66/64.5/56.5/54.5/48.5/48/]

[77.5/72.5/66/64.5/56.5/54.5/48.5/48/]

KCWL is 30597.5 and identification code is

488[77.5/72.5/66/64.5/56.5/54.5/48.5/48/]

For Fig. 3(b) the structural weighted link labels or labels are-

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The label and corresponding KCWL of both the figures are identical hence the chains are isomorphic.

Example-2:-Consider the two chains shown in Fig.4(a) and fig.4(b) (Nine links two degree of freedom kinematic chains,

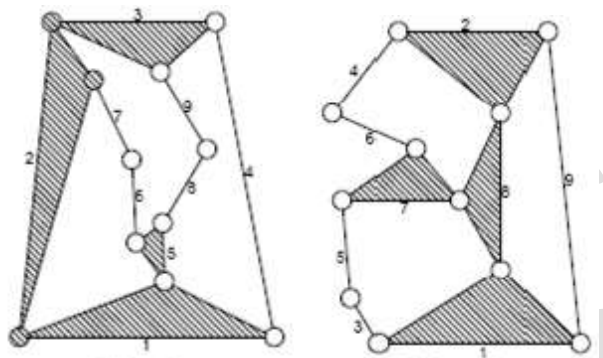


Fig.4(a)

Fig.4(b)

Nine links two degree of freedom kinematic chains

For chain shown in Fig.4(a), the labels, identification code , KCWL and distinct mechanisms are

Labels-/85/84.5/84/78.5/64.5/61/59/54.5/52/

Identification code-623[85/84.5/84/78.5/64.5/61/59/54.5/52]

KCWL-44620

Distinct mechanisms-9

For chain shown in Fig.4(b), the labels, identification code , KCWL and distinct mechanisms are-

Labels - /86/86/84/84/78/62.5/62.5/61/47/

Identification code- 651[86/86/84/84/78/62.5/62.5/61/47],

KCWL- 48730 and distinct mechanisms-6

The chain label and KCWL for nine links two degree of freedom chains shown in Fig.4(a) and Fig.4(b) are non identical, so it is clear from result that two chains are non – isomorphic.

Example-3 :- Three 12 links non-isomorphic kinematic chains are considered from ^(x) shown in Figures 5(a), 5(b) and 5(c) respectively. They have identical characteristic polynomial^(x). The labels and corresponding KCWL for the Figures are-

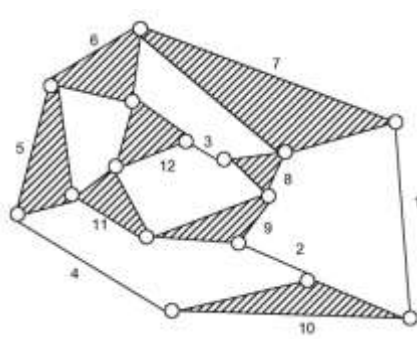


Figure 5(a)

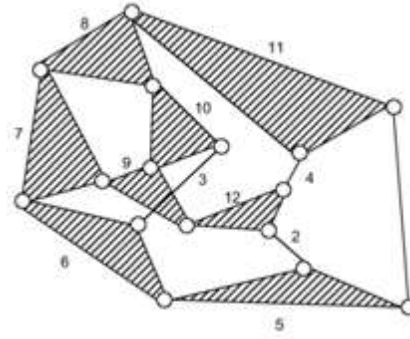


Figure 5(b)

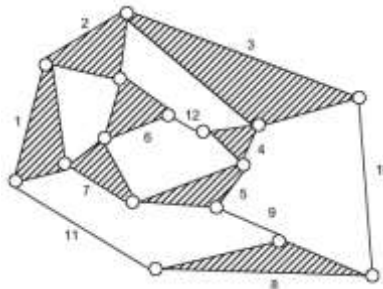


Figure 5(c)

Fig 5 Twelve-link 1-dof kinematic chains

For Figure 5(a)- /185/179/179/178/177/162/139/137/132/131/130/130/ and KCWL is 294039

For Figure 5(b) - /178/177/177/165/153/146/146/137/131/130/130/128/ and KCWL is 273722

And For Figure 5(c) /185/179/179/178/177/162/139/137/132/131/130/130/ and KCWL is 294039

Chain shown in Fig. 5(a) and 4(c) are isomorphic to each other, only links are relabeled in a different manner. Result clearly show that the invariant is independent of relabeling of links. KCWL for Fig 5 is different hence it clearly reflects that the chains are uniquely identified by KCWL.

Example 4:- Complete set of 8 links, 1 degree of freedom 16 kinematic chains are shown in Figure 6. The KCWL is calculated for each chain and shown in table-3 . All of these chains have distinct value of KCWL. The total distinct mechanism made by 8 links, 1 degree of freedom 16 kinematic chains are shown in same table.

RESULT AND CONCLUSIONS:-

Though no proof has been offered in the present work, but authors strongly believe that this method is unique and reliable as it takes care of nature and all inherent properties of the mechanism. In this paper a new heuristic method for detection of isomorphism among kinematic chains is presented. The proposed method is also tested to obtain all DM derived from a family of planar kinematic chain. It is hoped that the proposed method presents a new concept on which a new classification system for distinct mechanism and isomorphism of k-chains can be based. Method is simple, reliable and can easily be implemented on computer. A program is written in C++ and entire calculations was carried out in a personal computer with Pentium dual core ES200 @2.5 GH2 with 1 GB random access memory. With the help of present method kinematic chains can be checked with single numerical invariant. Also DM of a kinematic chain are derived from the family of 1 DOF 8 links 16 k chains in accordance is shown by table no-3.

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TABLE 3 ,(FIG.6) – KINEMATIC CHAIN LABELS AND DM OF EIGHT – LINK SINGLE DEGREE – OF – FREEDOM KINEMATIC CHAINS

S.NO.	K.CHAIN NO.	IDENTIFICATION CODE	KCWL	DISTINCT MECHANISMS
1	1	544[4(84)/4(52)]	39040	(1,4,5,8),(2,3,6,7)=2
2	2	492[4(73.5)/4(49.5)]	31410	(1,2,5,6),(3,4,7,8)=2
3	3	544[89.5/84.5/72.5/66/62.5/60.5/56.5/52]	38225	1,2,3,4,5,6,7,8=8
4	4	512[2(77.5)/2(71.5)/58/2(56.5)/43]	33834	(1,5),(2,4),3,(6,7),8=5
5	5	511[78.5/75.5/2(66.5)/2(61.5)/2(50.5)]	33372	(1,4),(2,8),(3,7),5,6,=5
6	6	487[70/(63)/4(61)/47]	29931	1,(2,6,7,8),(3,5),4=4
7	7	491[77.5/68.75/63.5/63/2(57.75)/63/40]	30398	1,2,3,4,5,6,(7,8)=7
8	8	488[74/73.5/72.25/66.25/59.75/56.5/43.75/42]	30927.75	1,2,3,4,5,6,7,8=8
9	9	503[2(79.5)/2(67)/2(56.5)/2(48.5)]	32707.5	(1,8),(2,7),(3,6),(4,5)=4
10	10	464[4(63)/4(53)]	27112	(1,3,4,7),(2,5,6,8)=2
11	11	481[2(68)/2(63)/62/2(55)/47]	29289	(1,3),(2,7),(4,6),5,8=5
12	12	504[4(66)/4(60)]	31824	(1,2,4,5),(3,6,7,8)=2
13	13	461[71.5/68.5/2(57.5)/55/53.5/52/46]	27124.25	1,(2,8),3,4,5,6,7=7
14	14	523[2(85)/2(69.5)/69/2(53.5)/38]	36048	1,(2,8),(3,7),(4,6),5=5
15	15	502[6(70.5)/2(39.5)]	32942	(1,4),(2,3,5,6,7,8)=2
16	16	452[4(64)/2(52)/2(46)]	26024	(1,4),(2,3,5,6),(7,8)=3

TOTAL DM-71

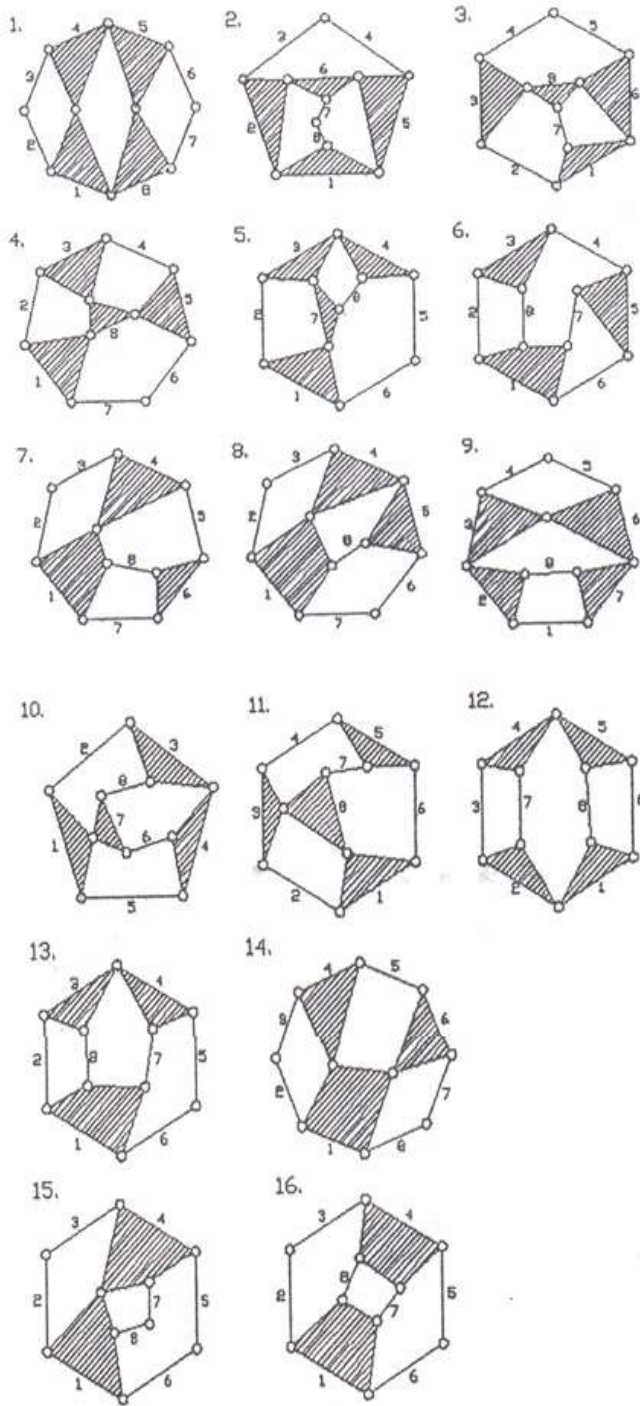


Figure-6, Eight links 1 degree of freedom 16 K-Chains