Design of Fuzzy C-Means Computed Torque Controller for 2-Link Flexible Robot Manipulator

Pydi Gowri Devi[1],Ch Ravi Kumar[2], Bala Murali Pydi[3]

1. PG student, Department of Electrical Engg., AITAM, Tekkali
2. Asso.Prof., Department of Electrical Engg., AITAM, Tekkali
3. Asso.Prof., Department of Electrical Engg., AITAM, Tekkali

Abstract: One of the important challenges that control engineers face in the field of robotics is manipulator control with acceptable performance. The main objective of this paper is to design a Fuzzy C-Means (FCM) controller for the position control of 2-link flexible robot manipulator. The robot manipulators are highly nonlinear, time variant and multiple input multiple outputs (MIMO) in nature. Computed Torque Controller (CTC) is an efficient nonlinear controller for controlling the position of robot manipulators. CTC works well when all the physical and dynamic parameters are known but when the robot manipulator has variation in dynamic parameters, and practically have large amount of uncertainties. Because of this reason, the controller has no acceptable performance. In order to overcome the disadvantages of CTC like not obtaining minimum error, fine trajectory, good disturbance rejection, a two input Fuzzy C-Means controller is proposed to control the position of the robot manipulator. Integration of this Fuzzy C-Means Proportional Derivative controller with Computed Torque Controller (CTC), and its application to two link flexible robot manipulator is also presented. The error in the each joint angle is also observed in this proposed work.In this paper the presented clustering algorithm allows us in classifying the data as distinct groups by using and/or functions. The optimal rule base for the proposed system in this paper is designed by using the clustering technique i.e. FCM technique. The optimal rule base is graphically obtained by using Phase Plane Analysis. The efficacy of the proposed controller is proved by comparing the results with results obtained with the normal conventional Computed Torque Controller (CTC).

Keywords: Flexible robot manipulator, CTC, FCM controller.

1. Introduction

The robotic applications are of wide range in field of engineering and technology. An important section of robot anatomy is the end manipulator. These manipulators are widely used in applications likewelding, assembling, painting, grinding, mechanical handling and other industrial applications. These applications may require exact path planning, trajectory generation and control design. The robot manipulators are highly nonlinear in nature. Computed Torque Controller (CTC) is powerful nonlinear controller which is widely used in controlling the position of robot manipulator. The main targets in designing control systems for robot manipulators are stability, good disturbancerejection, and small tracking error when all dynamic and physical parameters are known, computed torque controller works efficiently. But practically these systems have large amount of uncertainties. Therefore in this paper design of Fuzzy C-Means controller in integration with CTC, and its application to 2-link flexible robot manipulator is presented. The mathematical model of with computed torques is taken and is implemented in SIMULINK. This design of CTC is based on feedback linearization and computes the required arm torques using the nonlinear Feed-back control law. A non-classical method i.e. Mamdani type Fuzzy C-Means control is used here in order to obtain better results. The error in the angle is obtained by using equation e (θ) = θd- θa, where θd is desired position and θais the actual position of the manipulator. The obtained results are compared with that of the results obtained in the normal conventional Computed Torque Controller (CTC). The remaining part of the paper is organised as follows: section II explains about the mathematical model of 2-link flexible robot manipulator, Section III explains about design aspects of Fuzzy C-Means controller, Section IV explains about the simulation results and discussion.

2. Mathematical model of 2-link flexible robot manipulator:

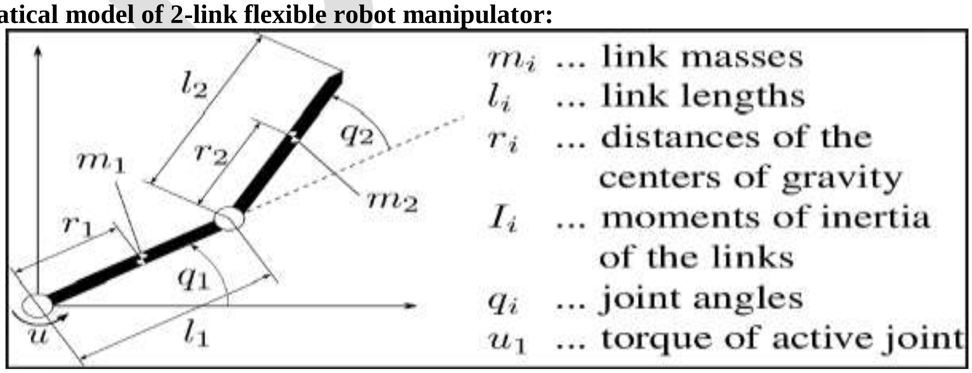

Figure 1: The two link flexible robot manipulator

The mathematical model with computed torques of the 2-link flexible robot manipulator is derived by using Lagrange's equation, which is given by [1] [2]

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q_i} = \tau_i$ ----- (1)

Where $L = K - P$ ----- (2)

$K = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$ ----- (3)

Thus, the kinetic energy for the link 1 with the linear velocity $v_1 = \frac{1}{2}L_1\dot{\theta}_1$, angular velocity $w_1 = \dot{\theta}_1$, moment of inertia $I_1 = \frac{1}{12}m_1{L_1}^2$, and mass is m_1.

$K_1 = \frac{1}{2}I_1{v_1}^2 + \frac{1}{2}I_1{w_1}^2 = \frac{1}{8}m_1{L_1}^2{\dot{\theta}_1}^2 + \frac{1}{24}m_1{L_1}^2{\dot{\theta}_1}^2 = \frac{1}{6}m_1{L_1}^2{\dot{\theta}_1}^2$ ----- (4)

And its potential energy is

$P_1 = \frac{1}{2}m_1 g L_1 sin\theta_1$ ------ (5)

Where g is the magnitude of acceleration due to gravity in the negative direction of Y-axis. For the second link, link 2, the Cartesian position coordinates (x_2, y_2) of the center of mass of link are:

$x_2 = L_1 cos\theta_1 + \frac{1}{2}L_2\, cos(\theta_1 + \theta_2)\,;\ y_2 = L_1 sin\theta_1 + \frac{1}{2}L_2 sin(\theta_1 + \theta_2)$ ----- (6)

Differentiating Equation (6) gives the components of velocity of link 2 as

$$\dot{x}_2 = -L_1 sin\theta_1\dot{\theta}_1 - \frac{1}{2}L_2\, sin(\theta_1 + \theta_2)\,(\dot{\theta}_1 + \dot{\theta}_2)$$

$\dot{y}_2 = -L_1 cos\theta_1\dot{\theta}_1 - \frac{1}{2}L_2\, cos(\theta_1 + \theta_2)\,(\dot{\theta}_1 + \dot{\theta}_2)$ ----- (7)

From these components, the sequence of the magnitude of velocity of the end of link 2 is

${v_2}^2 = {\dot{x}_2}^2 + {\dot{y}_2}^2$ or

${v_2}^2 = {L_1}^2{S_1}^2{\dot{\theta}_1}^2 + \frac{1}{4}{L_2}^2{S_{12}}^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + L_1L_2S_1S_{12}\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right) + {L_1}^2{C_1}^2{\dot{\theta}_1}^2 + \frac{1}{4}{L_2}^2{C_{12}}^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + L_1L_2C_1C_{12}\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right)$ ----- (8)

Simplifying

${v_2}^2 = {L_1}^2{\dot{\theta}_1}^2 + \frac{1}{4}{L_2}^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + L_1L_2C_2\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right)$ ----- (9)

Where $C_i = cos\theta_i,\ S_i = sin\theta_i,\ C_{12} = cos(\theta_1 + \theta_2)$ and $S_{12} = sin(\theta_1 + \theta_2)$ Thus the kinetic energy of link 2 with $w_1 = \dot{\theta}_1 + \dot{\theta}_2$and $I_2 = \frac{1}{12}m_2{L_2}^2$

$$K_2 = \frac{1}{2}m_2{v_2}^2 + \frac{1}{2}I_2{w_2}^2 = \frac{1}{2}m_2\left[{L_1}^2{\dot{\theta}_1}^2 + \frac{1}{4}{L_2}^2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 + L_1L_2C_2\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right)\right] + \frac{1}{24}m_2{L_2}^2(\dot{\theta}_1 + \dot{\theta}_2)^2$$

$= \frac{1}{2}m_2{L_1}^2{\dot{\theta}_1}^2 + \frac{1}{6}m_2{L_2}^2({\dot{\theta}_1}^2 + {\dot{\theta}_2}^2 + 2\dot{\theta}_1\dot{\theta}_2) + \frac{1}{2}m_2L_1L_2C_2\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right)$ ----- (10)

The potential energy of link 2, from Equation (6), is

$P_2 = m_2 g L_1 S_1 + \frac{1}{2}m_2 g L_2 S_{12}$ ----- (11)

The Lagrangian $L = K - P = K_1 + K_2 - P_1 - P_2$ is obtained and rearranging and simplifying, the Lagrangian is

$L = \frac{1}{2}\left(\frac{1}{3}m_1 + m_2\right){L_2}^2{\dot{\theta}_1}^2 + \frac{1}{6}m_2{L_2}^2\left({\dot{\theta}_1}^2 + {\dot{\theta}_2}^2 + 2\dot{\theta}_1\dot{\theta}_2\right) + \frac{1}{2}m_2L_1L_2C_2\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right) - \left(\frac{1}{2}m_1 + m_2\right)gL_1S_1 - \frac{1}{2}m_2 g L_2 S_{12}$ ----- (12)

From (2.3) the torque at joint1 is

$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \dot{\theta}_1}$ ----- (13)

The Lagrangian function L in (12) is differentiated w.r.t. θ_1 and θ_1 to give

$\frac{\partial L}{\partial \theta_1} = \left(\frac{1}{2}m_1 + m_2\right)gL_1C_1 - \frac{1}{2}m_2 g L_2 C_{12}$ ----- (14)

$\frac{\partial L}{\partial \dot{\theta}_1} = \left(\frac{1}{2}m_1 + m_2\right){L_1}^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2{L_2}^2\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \frac{1}{3}m_2L_1L_2C_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right)$ ----- (15)

Differentiating (15) w.r.t. time.

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \left[\left(\frac{1}{3}m_1 + m_2\right){L_1}^2 + \frac{1}{3}m_2{L_1}^2 + m_2L_1L_2C_2\right]\ddot{\theta}_1 + m_2\left[\frac{1}{2}{L_1}^2 + \frac{1}{2}L_1L_2C_2\right]\ddot{\theta}_2 - m_2L_1L_2S_2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}m_2L_1L_2S_2{\dot{\theta}_2}^2$ ----- (16)

Substituting the results obtained in (14) and (16) in (13), the torque at joint 1 is obtained as

$\tau_1 = \left[\left(\frac{1}{3}m_1 + m_2\right){L_1}^2 + \frac{1}{3}m_2{L_1}^2 + m_2L_1L_2C_2\right]\ddot{\theta}_1 + m_2\left[\frac{1}{2}{L_1}^2 + \frac{1}{2}L_1L_2C_2\right]\ddot{\theta}_2 - m_2L_1L_2S_2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}m_2L_1L_2S_2{\dot{\theta}_2}^2 + \left(\frac{1}{2}m_1 + m_2\right)gL_1L_2 + \frac{1}{2}m_2 g L_2 C_{12}$ ----- (17)

Similarly the derivatives of Lagrangian (12) for joint 2 are

$\frac{\partial L}{\partial \dot{\theta}_2} = \frac{1}{2}m_2L_1L_2S_2\left({\dot{\theta}_1}^2 + \dot{\theta}_1\dot{\theta}_2\right) - \frac{1}{2}m_2 g L_2 C_{12}$ ----- (18)

$$\frac{\partial L}{\partial \dot{\theta}_2} = \frac{1}{3} m_2 {L_2}^2 (\dot{\theta}_1 + \dot{\theta}_2) + \frac{1}{2} m_2 L_1 L_2 C_2 \dot{\theta}_1$$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \left[\frac{1}{3} m_2 {L_2}^2 + \frac{1}{2} m_2 L_1 L_2 C_2\right] \ddot{\theta}_1 + \frac{1}{3} m_2 {L_2}^2 \ddot{\theta}_2 - \frac{1}{2} m_2 L_1 L_2 S_2 \dot{\theta}_1 \dot{\theta}_2$ ----- (19)

So that torque at joint 2

$\tau_2 = \left[\frac{1}{3} {L_2}^2 + \frac{1}{2} L_1 L_2 C_2\right] \ddot{\theta}_1 + \frac{1}{3} m_2 {L_2}^2 \ddot{\theta}_2 - \frac{1}{2} m_2 L_1 L_2 S_2 {\dot{\theta}_1}^2 + \frac{1}{2} m_2 g L_2 C_{12}$ ----- (20)

Equations (17) and (20) are the EOM (dynamic model) of the 2-link planar manipulator. Because both the joints are revolute, the generalized torques t_1 and t_2 represent the actual joint torques.

From (17) and (20), the generalized torque equation can be written as:

$\tau_1 = M_{11} \ddot{\theta}_1 + M_{12} \ddot{\theta}_2 + H_1 + G_2$

$\tau_2 = M_{21} \ddot{\theta}_1 + M_{22} \ddot{\theta}_2 + H_2 + G_2$ ----- (21)

Where

$$M_{11} = \left[\left(\frac{1}{3} m_1 + m_2\right) {L_1}^2 + \frac{1}{3} m_2 {L_2}^2 + m_2 L_1 L_2 C_2\right]$$

$$M_{12} = M_{21} = m_2 \left[\frac{1}{3} {L_2}^2 + \frac{1}{2} L_1 L_2 C_2\right]$$

$M_{22} = \frac{1}{3} m_2 {L_2}^2$; $H_1 = -m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 - \frac{1}{2} m_2 L_1 L_2 {\dot{\theta}_2}^2$

$$H_2 = \frac{1}{2} m_2 L_1 L_2 {\dot{\theta}_1}^2$$

$$G_1 = \left[\left(\frac{1}{2} m_1 + m_2\right) L_1 C_1 + \frac{1}{2} m_2 L_2 C_{12}\right] g$$

$$G_2 = \frac{1}{2} m_2 L_2 C_{12} g$$

3. Design of Fuzzy C-Meanscontroller:

3.1 Design procedure

In the present practice, fuzzy logic technique [3] [4] is an emerging research area due to its application to complex systems is very much successful, where some conventional methods like PID controllers are difficult to apply. The Fuzzy Logic model is empirically-based, relying on an operator's experience rather than their technical knowledge of the system [5] [6]. Fuzzy C-Means (FCM) is a clustering [7] method which allows one piece of data belong to two or more clusters. The design methodology of FCM controller is as follows:[8]

Step 1: The Normal Fuzzy controller is designed heuristically with fuzzy linguistic rules.
Step 2: The Fuzzy C-Means controller is tuned to the normal fuzzy controller.
Step 3: The phase-plane plot of the input space is obtained.
Step 4: The input space is divided into clusters using Fuzzy C-Means and the cluster centers are identified.
Step 5: The sequence of rules of the original fuzzy controller is super imposed onto the phase-plane plot of the input space with cluster centers.
Step 6: Hence the required rules are identified and the non-cooperative rules are thus eliminated.

$J_m = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m \left\| x_i - c_j \right\|^2$ ----- (22)

where m is any real number greater than 1, u_{ij} is the degree of membership of x_i in the cluster j, x_i is the i^{th} of d-dimensional measured data, c_j is the d-dimension center of the cluster [15] [16], and ||*|| is any norm expressing the similarity between any measured data and the center.

Fuzzy partitioning is carried out through an iterative optimization of the objective function shown above, with the update of membership u_{ij} and the cluster centers c_j by:

$u_{ij} = \frac{1}{\sum_{j=1}^{C} \left(\frac{\|x_i - c_j\|}{\|x_i - c_k\|}\right)^{\frac{2}{m-1}}}$ where $C_j = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m}$ ----- (23)

This iteration will stop when $\left\{\left| u_{ij}^{(k+1)} - u_{ij}^{k} \right|\right\} < \in$, where $\in$ is a termination criterion between 0 and 1, whereas k is the iteration step [9]. This procedure converges to a local minimum or a saddle point of Jm. In a batch mode operation, FCM determines the cluster centers c_i, and the membership matrix U using the following steps [10] [11]:

Step 1: Set the number of clusters c. Initialize the membership matrix U with random values between 0 and 1 such that the summation of degrees of belongingness of a data point to all clusters is always equal to unity.

Step 2: Calculate c Fuzzy cluster centers, 1, c_i where i = 1, 2...c, using eq. (3).

Step 3: Compute the objective function according to eq. (2). Stop if either it is below a certain tolerance value or its improvement over previous iteration is below a certain threshold.

Step 4: Compute a new U using eq. (3). Go to Step 2.

3.2 Phase Plane plots

The Fuzzy C-Means model is an empirical based, relying on an operator's experience rather than their technical knowledge of the system [12]. And the major drawback is the design of the rule base. By using the phase-plane plots for the given to the fuzzy controller rule base is obtained. Stability of fuzzy system requires characterization of the relation between the rule base and state space with the dynamic system under control.This relation is based on the relative of influence of every rule of the rule base by fuzzy inference engine. A closed loop trajectory can be mapped on the position of the space. A sequence of rules obtained according to the order in which they are fired forms the solution called linguistic trajectory. This provides guidelines to obtain the necessary rule base from the phase plane plots of the inputs given to the fuzzy controller. The clusters are formed in entire position space of the inputs using Fuzzy C-Means. The cluster centers are identified and marked on the phase-plane plot [13] [14]. These plots are mapped with the closed–loop trajectory and the required.

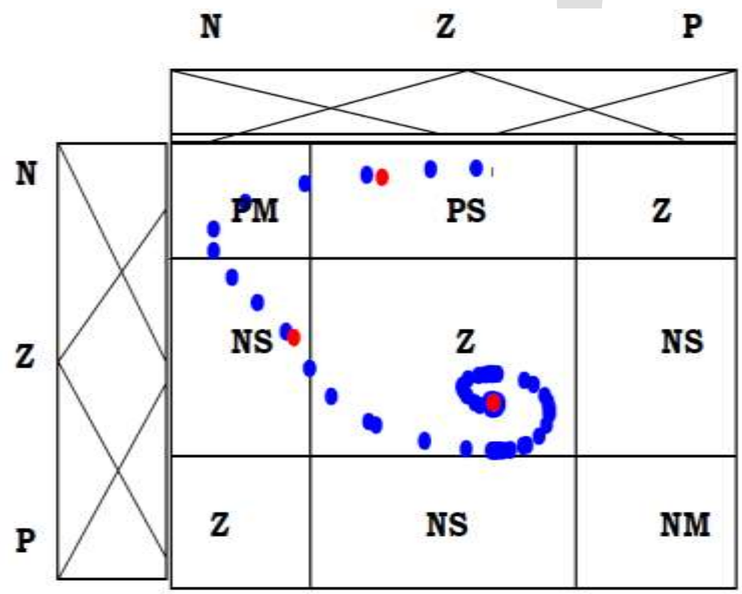

Figure 2: Phase Plane plot

Table 1 The effective rule base

Input 1	PB	NS	NS	NB	PS
Input 2	PS	PB	NS	NB	NB
Output	NB	NS	PM	PB	NM

4. Results and Discussions

The proposed Fuzzy C-Means controller applied to 2-link flexible robot manipulator has been tested for both step and ramp inputs and compared with normal CTC and reference signal. Therobotic arm mathematical model with computed torques was simulated to evaluate the performance of the controller. The difference between the desired location and current location is an input vector to the controller that generates joint rate commands. The performance of the proposed controller is tested in the presence of uncertainties such as inertial and gravitational constants. The results presented in this paper prove the effective performance of the proposed controller. The development of the control algorithm, simulation, testing, results, and the performance of the controller are reported. From Figures (3-8) it can be observed that the responses of theta value with Fuzzy C-Means controller when both ramp and step inputs are given. The numerical data analysis is shown in Tables (2-6). The similar analysis was done for the remaining three links. The minimized fuzzy rule base is shown in Table 1. This work concludes that the Fuzzy C-Means based controller outperformed the other controllers. The peak time, delay time, rise time, settling time and the peak overshoot are reduced considerably.

Conclusions:

Here in this article a novel approach, designing of Fuzzy C-Means Controller (FCM) is presented. It is a fuzzy rule based approach for robot motion control to eliminate the computational complexity associated with the conventional mathematical algorithm. The errors in the joint angles of manipulator are minimized considerably. In this paper fuzzy computed torque controller with minimum rules is obtained by validating the clusters to choose most contributed rules. The fuzzy clustering technique in addition with the phase-plane plot of the inputs of the fuzzy controller is utilized and finally required rules are identified, the non-cooperative or unfired rules are thus eliminated. The numerical analysis shows the effectiveness of the proposed FCM controller in minimizing the error in joint angles when compared to Computed Torque Controller (CTC) and that of the reference signal.

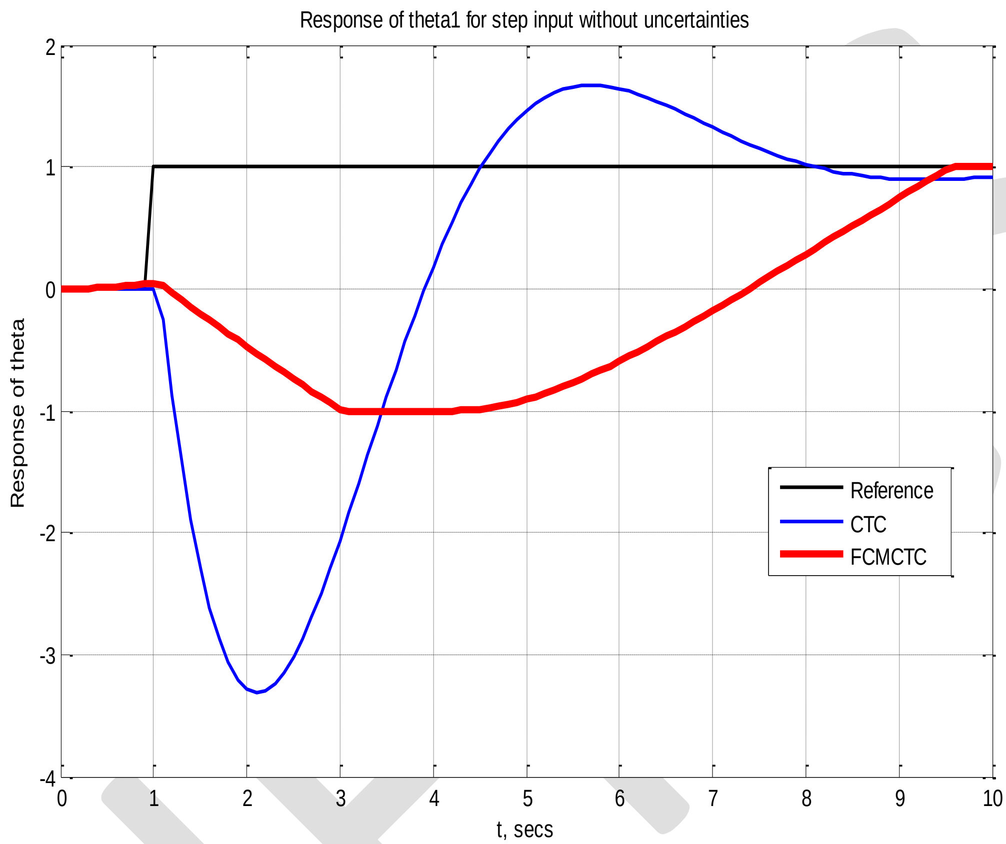

Figure 3: Response of thta1 for step input without uncertainties

Table 2: Response of thta1 for step input without uncertainties

Control technique	T_p(sec)	T_d(sec)	T_s(sec)	T_r(sec)	M_p	e_{ss}
CTC	5.89	4.5	12.3	5.05	0.655	0.21
FCM	9.41	6.51	9.41	7.85	0	0

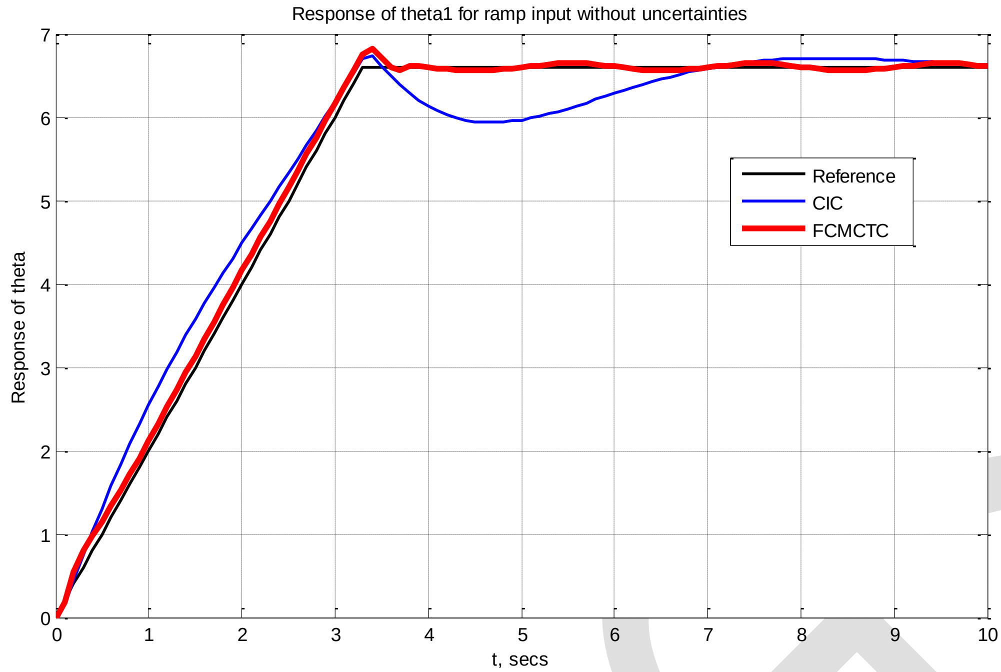

Figure 4: Response of thta1 for ramp input without uncertainties

Table 3: Response of thta1 for ramp input without uncertainties

Control technique	T_p (sec)	T_d (sec)	T_s(sec)	T_r(sec)	M_p	e_{ss}
CTC	3.4	1.75	8.95	2.982	0.241	0.21
FCM	3.23	1.58	3.5	2.935	0.225	0.04

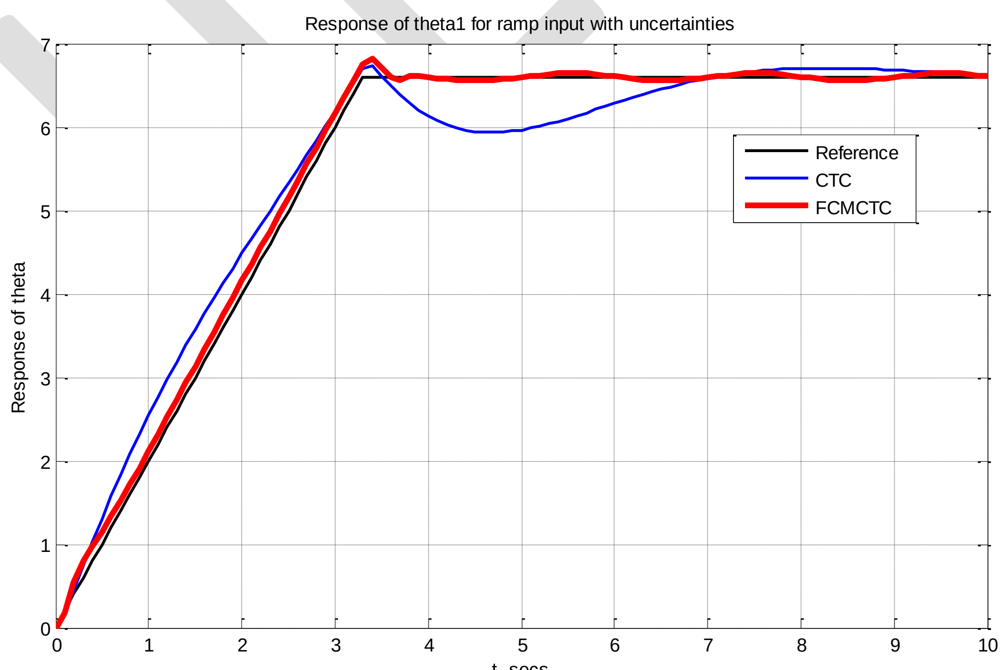

Figure 5: Response of thta1 for ramp input with uncertainties

Table 4: Response of thta1 for ramp input with uncertainties

Control technique	T_p (sec)	T_d (sec)	T_s(sec)	T_r(sec)	M_p	e_{ss}
CTC	3.4	1.75	8.95	2.982	0.241	0.21
FCM	3.23	1.58	3.5	2.935	0.225	0.04

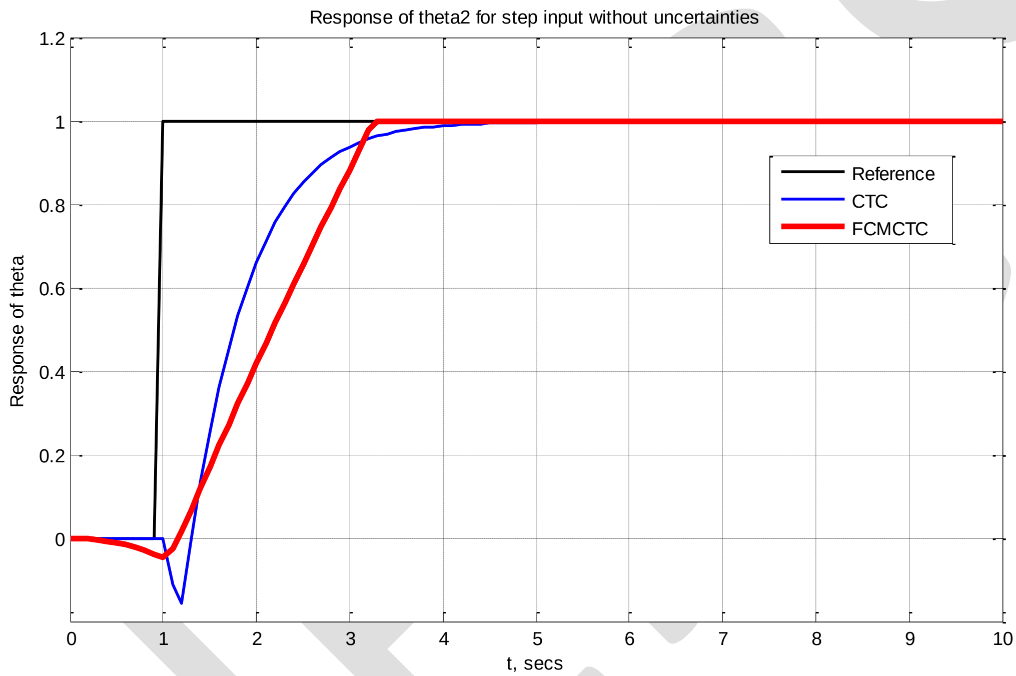

Figure 7: Response of thta2 for step input without uncertainties

Table 5: Response of thta2 for step input without uncertainties

Control technique	T_p (sec)	T_d (sec)	T_s(sec)	T_r(sec)	M_p	Control technique
CTC	3.74	1.68	4.53	2.82	-0.157	CTC
FCM	3.17	2.08	3.21	2.94	-0.043	FCM

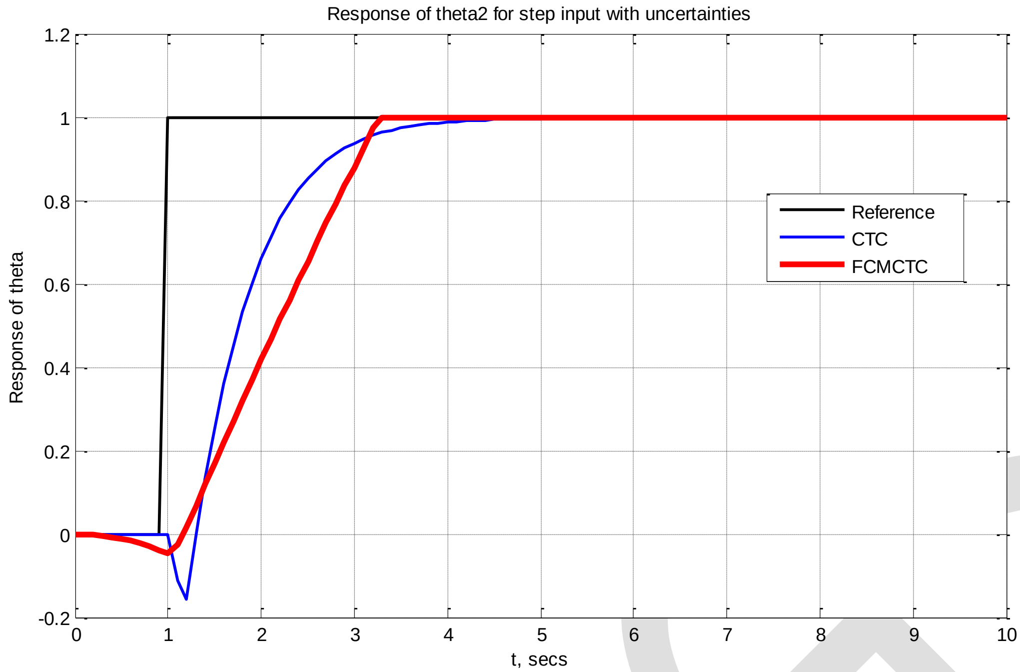

Figure 8: Response of thta2 for step input with uncertainties

Table 6: Response of thta2 for step input with uncertainties

Control technique	T_p (sec)	T_d (sec)	T_s(sec)	T_r(sec)	M_p	M_p
CTC	3.74	1.68	4.53	2.82	-0.157	-0.157
FCM	3.17	2.08	3.21	2.94	-0.043	-0.043

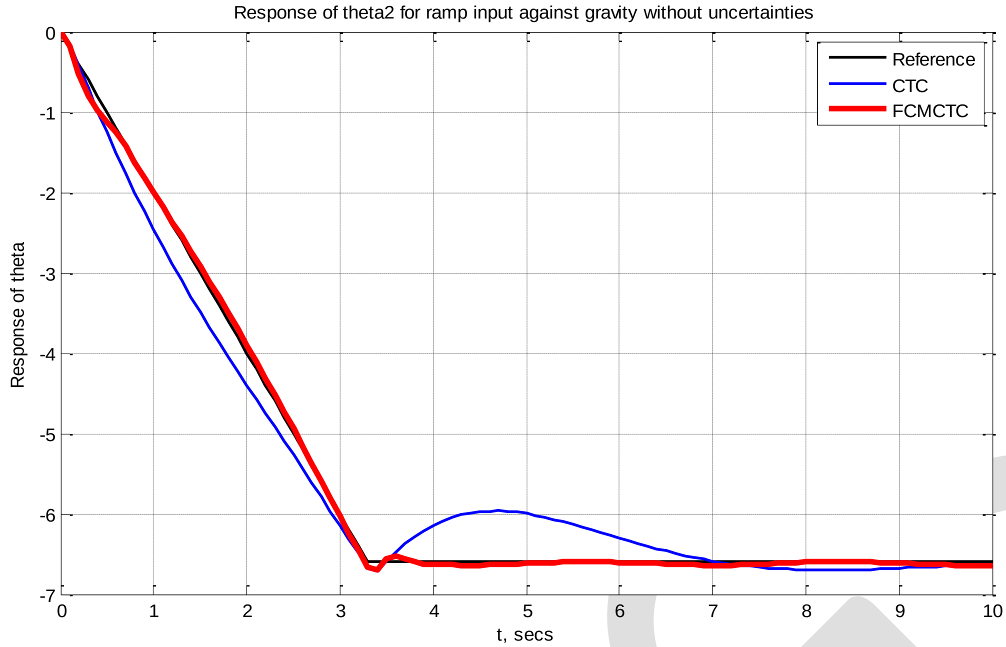

Figure 9: Response of theta2 for ramp input against gravity without uncertainties

Table 7: Response of theta2 for ramp input against gravity without uncertainties

Control technique	T_p(sec)	T_d(sec)	T_s(sec)	T_r(sec)	M_p	e_{ss}
CTC	3.4	1.74	6.6	3.1	0.205	0.22
FCM	3.35	1.578	3.75	2.96	0.125	0.0075

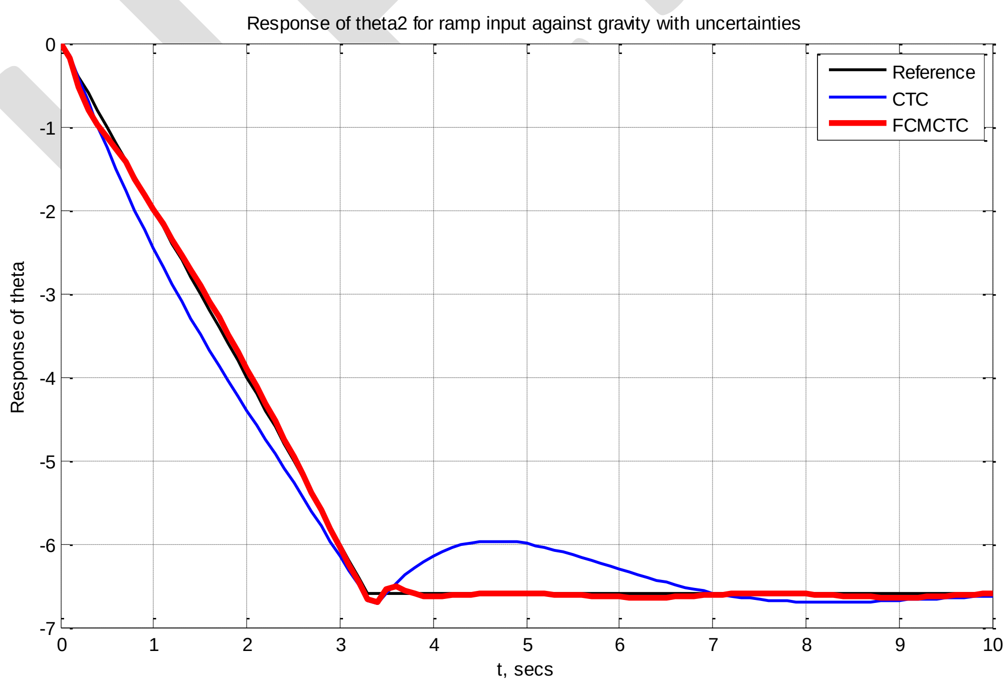

Figure 10: Response of theta2 for ramp input against gravity with uncertainties

Table 8: Response of theta2 for ramp input against gravity with uncertainties

Control technique	T_p(sec)	T_d (sec)	T_s(sec)	T_r(sec)	M_p	e_{ss}
CTC	3.4	1.74	6.6	3.1	0.205	0.22
FCM	3.35	1.578	3.75	2.96	0.125	0.0075

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