Multiple Vehicles Detection using MVDR based Phased Array Radar Beamformer

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Abstract— To achieve 'Vision Zero Collision' objective in smart vehicle, radar based ADAS (Automatic Driving Assistant System are being developed worldwide. Motivated by the above requirement toward public safety driving, this paper aims at modeling such radar for vehicular friendship. This paper is now catering for development and modeling of radar having two ADAS modes of operation namely i) ADAS1 with mode 1 having capability of collision avoidance by detecting the multiple target present on the road with the help of switched transmit beam like Minimum Variance Distortionless Response (MVDR).

For today's vehicular driver assistance, this digital beam forming based adaptive tracking system is highly efficient and approaching towards cost effective commercial utilization.

Index Terms— ADAS (Automatic Driving Assistant System), MVDR (Minimum Variance Distortionless Response).

INTRODUCTION

As smart RADAR continues to spread throughout the Intelligent Transport System (ITS) industry globally, it is essentially needed to find comforts for the drivers where they could be relaxed during a long drive through the transport highways. To support the driver by means of Automatic Cruise Control after a successful tracking is achieved by the switched beam technology (i.e., MVDR based target detection), a large scale efforts have been executed throughout the world in building the "smart car". The minimum variance distortionless response (MVDR) beamformer and its associated adaptive algorithm, the generalized sidelobe canceller (GSC), are probably the most widely studied and used beamforming algorithms, and are basis to some commercially available arrays. Assuming the direction of arrival (DOA) of the desired signal is known, the MVDR beamformer estimates the desired signal while minimizing the variance of the noise component of the formed estimate. In practice, however, the DOA of the desired signal is not known exactly, which significantly degrades the performance of the MVDR beamformer. A lot of research has been done into a class of algorithms known as robust MVDR. As a general rule, these algorithms work by extending the region where the source can be located. Nevertheless, even assuming perfect sound source localization (SSL), the fact that the sensors may have distinct, directional responses adds yet another level of uncertainty that the MVDR beamformer is not able to handle well. Commercial arrays solve this by using a linear array of antenna elements, all pointing at the same direction, and therefore with similar directional gain.[4]

In this paper, the Basics of Beamforming has been illustrated under Section-II. The Linear Array Design part has been discussed in Section-III. Mathematical algorithm of "Direction of Arrival Estimation" has been given under Section-IV. The Section-V deals with how the multiple beam formations have been executed to achieve Switched Beam technique. The details of Minimum Variance Distortionless Response (MVDR) based Beamformer have been pointed out in Section-VI. The MUSIC Algorithm as a special technique for Direction of Arrival Estimation is discussed under Section-VII. The working simulation model and its Block Diagram has been illustrated in Section-VIII. The final outcomes of the model have been framed under Section-IX. The Conclusion of the designed MVDR based Phased Array Radar Beamformer has been highlighted under Section-X.

BASICS OF BEAM-FORMING

Beamforming is an advanced signal processing technique which, when employed along with an array of transmitters or receivers will be capable of controlling the 'directionality of' or 'sensitivity to' a particular radiation pattern. This method creates the radiation pattern of the antenna array by adding the phases of the signals in the desired direction and by nullifying the pattern in the unwanted direction. The inter element phase usually adjusts the amplitudes to optimize the received signal. A standard tool for analyzing the performance of a beamformer as shown in Fig.1. In Fig. 1 the outputs of the individual sensors are linearly combined after being scaled with the corresponding weights optimizing the antenna array to have maximum gain in the direction of desired signal and nulls in the direction of interferers. For beamformer the output at any time n , y(n) is given by a linear combination of the data at M antennas, with x(n) being the input vector and w(n) being the weight vector.[1][8]

Fig. 1: Beamforming

$y(n) = w^{H}(n) * (n)$....... (1)

Weight vector W (n) can be define as:

$w(n) = \sum_{n=0}^{M-1} wn$......... (2)

And

$x(n) = \sum_{n=0}^{M-1} Xn$.......... (3)

For any algorithm that avoids matrix inverse operation and uses the instantaneous gradient vector $\nabla J(n)$ for weight vector up gradation the weight vector at time n + 1can be written as:

$\mathbf{W}(n+1) = W(n) + \frac{1}{2}\mu[\nabla J(n)]$.......... (4)

Where µ is the step size parameter, which controls the speed of convergence and it lies between 0 and 1. Very small values of µ leads to the slow convergence and good approximation of the cost function; on the contrary the large values of µmay lead to a faster convergence but the stability around a minimum value may be lost.[8]

$0 < \mu < \frac{1}{\lambda}$.......... (5)

An exact calculation of instantaneous gradient vector $\nabla J(n)$ is not possible as prior information of covariance matrix R and cross-correlation vector p is needed. So an instantaneous estimate of gradient vector $\nabla J(n)$.

$\nabla \mathbf{J}(\mathbf{n}) = -2p(n) + 2R(n)W(n)$......... (6)

$\mathbf{R}(\mathbf{n}) = X(n)X^{H}(n)$......... (7)

$\mathbf{P(n)} = d^*(n)X(n)$……….. (8)

By putting values from (6, 7, and 8) in (4) the weight vector is found to be

W(n+1) = $\mathbf{W(n)} + \mu[\mathbf{p(n)} - \mathbf{R(n)W(n)}]$

$= \mathbf{W(n)} + \mu\mathbf{X(n)}[d^*(\mathbf{n}) - X^H(\mathbf{n})\mathbf{W(n)}]$

$= \mathbf{W(n)} + \mu\mathbf{X(n)}e^*(\mathbf{n})$

The desired signal can be define by three equations below:

$$y(n) = w^H(n) * (n)$$

$$\mathbf{e(n)} = d(n)y(n)$$

$$\mathbf{W(n+1)} = \mathbf{W(n)} + \mu\mathbf{X(n)}e^*(\mathbf{n})$$

LINEAR ARRAY DESIGN

To electronically scan a radiation pattern in a given direction, it is essential to have an array of elements arranged in a specific configuration. Although linear arrays lack the ability to scan in 3-D space, the planar array scan the main beam in y direction of θ (elevation) and φ (azimuth). Following the design of the individual rectangular patch antenna, a linear array of eight micro strip patches with inter element spacing of λ/2 (half wavelength), where space in cm is based on the resonance frequency.[6]

The reasons for choosing inter element spacing of λ/2 are as follows: To combat fading, the inter element spacing of at least λ/2 is necessary so that the signals received from different antenna elements are (almost) independent in a rich scattering environment (more precisely, in a uniform scattering environment). In such cases, the antenna arrays provide performance improvement through spatial diversity. However, to avoid grating lobes (multiple maxima), the inter element spacing should not exceed one wavelength. However, to avoid aliasing and causing of nulls to be misplaced, the inter element spacing should be less or equal to λ/2 (the Nyquist rate). Thus, to satisfy all three conditions, the inter element spacing of λ/2 (half wavelength) is chosen.

The total amplitude radiation patterns of the 16-element linear array based on the cavity model are represented, neglecting coupling, by the product of the element pattern (static pattern) and the array factor (dynamic pattern).

DIRECTION OF ARRIVAL ESTIMATION

The objective of this part is to find the direction of the desired user is and t the interference in order to steer the main beam towards the desired user and null along the interference. We have used a linear array here. Assuming this array to be equispaced and lay out along with the z-axis and the array factor is:

$$R(\theta) = \sum_{m-}^{M} -N\frac{Ic}{I_0}\exp(j\, n\, k \cos[\theta)]$$

In is the complex currents that are fed to array element and Ic is the reference current. Total number of elements are 2N+1 and d is the inter element spacing of the array. To estimate the direction of arrival of a signal that impinges on the array elements, suppose a source lies at a distance that is much greater than the separation between the array elements. The induced currents in the array elements will be same, but with a successive phase the difference given by

$$\epsilon = kd\ cos\theta$$

θ is angle of direction of arrival of signal with the axis of array as shown in Fig. 2.

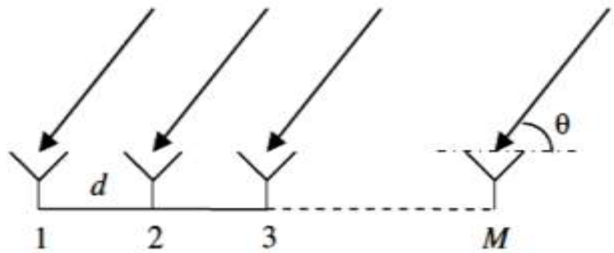

Fig. 2: Linear Array

Without loss of generality, assuming the azimuth of all incident signals on antenna array are $\frac{\lambda}{2}$, that is, all signals are from the same plane, the elevation angle of ith incident signal is i^{th} . The coordinate vector of the mth antenna in space is:

$$u_m = [[X]_m, y_m, Z_m] = [0, (m-1)d, 0]$$

Then, the signal steer vector on array elements can be expressed as Equation steering [13]

$$R_\theta = \begin{bmatrix} 1 \\ \exp\left\{j\frac{2\pi}{\lambda}(sin\theta_1)d\right\} \\ \exp\left\{j\frac{2\pi}{\lambda}(sin\theta_1)(m-1)d\right\} \end{bmatrix}$$

METHOD

This paper presents a single target and radar with 16-element antenna array. Assume that there are d desired signal sources and i interference sources simultaneously transmitting on same frequency channel, initially the antenna receives signals from different sources (user) and each element of antenna have incident wave at the same time each user have angle. Next the weight (w) will get many values (amplitude, phase). The w selects the beiger value to steer beam (desired) for the user by using the following received signal equation. [5]

$$X(n) = \sum_{j=1}^{d} a(0)s(t) + \sum_{k=1}^{1} a(\theta_{int-k})s_k(t)$$

Where $a(\theta)$ are array steering vector denoting the amplitude gain and the phase shift of the signal at the i-th antenna relative operator. s (t) is signal. The beamformer system output can be written as:

$$y(n) = w^H(n) * (n)$$

Then the null-steering beam-forming problem can be formulated as:

1. Estimation DOAs by using algorithm:

There are a set of methods, which are used to estimate DOA such as the Multiple Signal Classification (MUSIC) method is the first of the high-resolution algorithms for correcting the underlying data model of narrow band signals in additive noise. The next algorithm is Minimum Norm-Method (MNM) and ESPRIT using to find DOAs. An expression for the power spectrum is given by,

$$P(\theta) = 1/(a_{\downarrow}\theta^{\dagger}H\ R^{\dagger}(-1)\ a_{\downarrow}\theta\ )\ P(\theta)$$

$$= \frac{1}{a_\theta^H R^{-1} a_\theta}$$

Where: R array correlation matrix.

The estimated DOAs are to be classified for this type of validation process which separate out desired signal from

Interferences are carried out. After the validation process if, it is found that there is only single desired user and others

are interferences then the null steering mode is invoked. If it is found that there is more than one desired user then multi-beam-forming mode is invoked. The optimization of two vectors comprises of two techniques

- MVDR/ CAPON
- LCMV

MINIMUM VARIANCE DISTORTIONLESS RESPONSE (MVDR) BEAMFORMER

This is the case in which the signal is nonrandom but unknown. In the initial discussion, we consider the case of single plane-wave signal. [2]

The frequency-domain snapshot consists of signal plus noise,

$$X(\omega) = X_s(\omega) + N(\omega) \ldots\ldots\ldots\ldots (1)$$

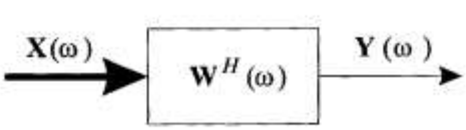

Fig. 3 Matrix processor

The signal vector can be written as

$X_s(\omega) = F(\omega)v(\omega{:}K_s)$, ……….. (2)

Where, $F(\omega)$ is the frequency domain snapshot of the source signal and v(cj:ks) is the array manifold vector for a plane wave with wave number ks. The noise snapshot, N (u>) is a zero-mean random vector with spectral matrix,

$S_n(\omega) = S_c(\omega) + \sigma_\omega^2 L$ ………….. (3)

Later we will add the assumption $N(\omega)$ that is a zero-mean circular complex Gaussian random vector.

We process $X(\omega)$ with a matrix operation $W^H(\omega)$ as shown in Figure 3. The dimension of $W^H(\omega)$ is 1xN.

The first criterion of interest is called the distortionless criterion. It is required that, in the absence of noise,

$Y(\omega) = F(\omega)$……….. (4)

For any $F(\omega)$. Under this constraint, we wish to minimize the variance of $Y(\omega)$ in the presence of noise. Thus, we write

$Y(\omega) = F(\omega) + Y_n(\omega)$……… (5)

and minimize $[E[|Y_n(\omega)|]^2]$

The constraint of no distortion implies

$W^H(\omega)v(\omega{:}K_s) = 1$ ………. (6)

The mean square of the output noise is,

$[E[|Y_n|]^2] = W^H(\omega)S_n(\omega)W(\omega)$……….. (7)

We want to minimize $[E[|Y_n|]^2]$ subject to the constraint in (6)

This second criterion is called the minimum variance unbiased estimate criterion. Here we require that $Y(\omega)$ be the minimum variance unbiased estimate of $F(\omega)$. This implies [2]

E[Y(ω)]= $F(\omega)$…… (8)

Thus, using (5) and (8),

$$E[Y(\omega)] = E[F(\omega)] + E[N(\omega)]$$

$$= E[F(\omega)]$$

For any $F(\omega)$. This equality implies

$$W^H(\omega)v(\omega : K_s) = 1$$

which is identical to the constraint in (6).

MUSIC ALGORITHM

While several algorithms for AOA estimation are available using the measurements S_i or S_{ij} (e.g. MUSIC [4]), our goal is to obtain general design guidelines for the antennas used at the radar receiver. Analysis in this section adopts the Fisher information (F) [5], [6] to evaluate the accuracy of a generic AOA estimation algorithm in relation to some important design parameters. The value F measures the amount of information that a signal carries about an unknown parameter. In the case discussed, the signals are the S_{ij} values, while the parameter to estimate is θ. The Fisher information is [7]

$$F_{ij}(\theta) = E\left\{\left[\frac{\partial}{\partial\theta}\log f_{ij}\left(s_{ij};\theta\right)\right]^2\right\}, \ldots\ldots\ldots. (1)$$

Where $f_{ij}\ (s_{ij});\theta)$ is the probability density function (pdf) that describes the measurements S_{ij}. Assuming independent and identically distributed (i.i.d) noise components N_{ij} with Gaussian distribution $N(0, \sigma_{rss}^2)$, it is possible to demonstrate that the total amount of Fisher information available from a system with N_r antennas is

$$F(\theta) = \frac{1}{\sigma_{rss}^2}\sum_{\{i,j\}\in C}\left[\frac{\partial\left(G_i - G_j\right)}{\partial\theta}\right]^2$$

$$= \frac{1}{\sigma_{rss}^2\left[N_r\sum_{i=0}^{N_r}\left(\left[\frac{\partial G_i}{\partial\theta}\right]\right)^2 - \left(\sum_{i=0}^{N_r-1}\left[\frac{\partial G_i}{\partial\theta}\right]\right)^2\right]} \ldots\ldots\ldots\ldots (2)$$

Where C denote the set of the Nr(Nr −1)/2 distinct antenna pairs {i, j}

- AOA Estimation Bound

The Fisher information F(θ) expresses the theoretical limit achievable when estimating θ. If T is an unbiased estimator for θ, the inverse of F(θ) bounds the minimum variance of the estimation error

$$Var[T(X)] \geq F^{-1}(\theta) \ldots\ldots\ldots (3)$$

The inequality, known as the Cram´ er-Rao Bound (CRB) [2], is a limit that applies to any estimator that uses measurements Sij to compute θ. The Fisher information and the CRB depend on the gains Gi of the antennas used in the system. To obtain general design guidelines, we model the gain of a generic patch antenna using a cardioid shaped function with exponent m ≥ 1, and maximum gain Gmax [7]

$$G(\theta) = G^{m}_{\max}\left(\frac{1+\cos(\theta)}{2}\right) = G^{2m}_{\max}\left(\cos\left(\frac{\theta}{2}\right)\right)$$ .... (4)

As shown in Fig. 2a, larger values of the exponent m correspond to more directive antennas. Substituting (4) into (2), we obtain an analytical expression for the CRB

$$N_r \sum_{i=0}^{N_r} [\tan\left(\frac{\theta - i\Delta}{2}\right)^2 - \left(\left(\sum_{i=0}^{N_r-1} \tan\left(\frac{\theta - i\Delta}{2}\right)\right)^2]\right)$$ …….. (5)

Where $CRB(\theta) = F^{-1}(\theta)$, and Δ = 360◦/Nr. Figure 2b shows the CRB for a system with $N_r = 2$ faces and m= 2. Note the different values of the CRB for different θ values. The minimum error is achieved for angles θ= 0◦and 180◦, while larger estimation error are to be expected for angles close to θ=±90◦. The error can be understood by observing the function that describes the gain difference between two equispaced faces (see Fig. 2c). The function G_1 (θ)–G_2 (θ) changes abruptly for angles close to 0◦and180◦: small variations of the angle determine large variation in the gains' difference. Since the Fisher information depends on the derivative of G_1 (θ)– G_2 (θ), this condition corresponds to a lower estimation error for θ. On the other hand, the estimation error increases for angles close ±90◦, where G_1 (θ)– G_2 (θ) has a less steep slope.

BLOCK DIAGRAM

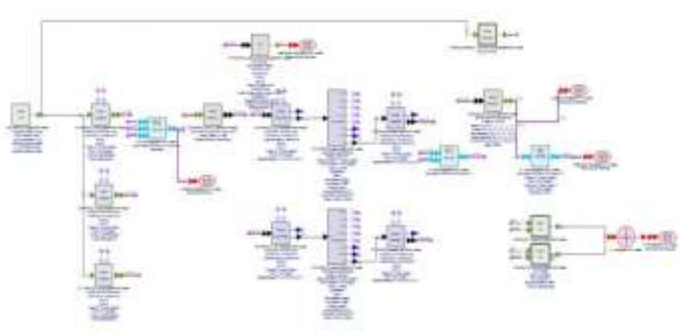

Fig. 4 Block Diagram to generate Switched Beam

The above block diagram includes:

- **LFM** waveform generator which will generate Linear Frequency Modulation.
- **Radar_Tx_DBS_2D** (Digital Beam Synthesis) this model is used to synthesize the main lobe along the direction determined by theta and phi.
- **Math Lang** (Math Language Model) model uses Math Language equations to process input data and produce output data.

- **Radar_MultiCH_Tx** this model is the behaviour simulation model of multi-channel transmitter. It's a timed model which is used to represent analog/RF circuit digital implementation that involves the notion of time in its behaviour.
- **Radar_Tx_Synthesis** this model is used to synthesis the electric wave from plane array to far field observation point. Both rectangle array of equally spaced elements and user defined phase shift are supported. xi is the ith input signal, y(t) is the output signal, Ai is the phase to be shifted for the ith array element, N is the number of input signals.

$$y(t) = \sum_{i=1}^{N} x_i(t) * A_i(t)$$

- **Radar_TargetEcho** this model is to generate the echo of the moving target under the Cartesian and Spherical coordinate. The typical RCS values of kinds of targets are given by the parameter. The user defined RCS values is support too.
- **Radar_PhaseShift** this model is used to add phase shift from the far electric field observation point to digital array antenna. Both rectangle array of equally spaced elements and user defined phase shift are supported. X(t) is the input signal, yi(t) is the ith output signal, Ai is the ith phase to be shifted, then

$$yi(t) = x(t) * \exp(j * Ai)$$

- **Radar_MultiCH_Rx** this model is the behaviour simulation model of multi-channel receiver from RF to baseband. The model input is a timed input which is used to represent analog/RF circuit digital implementation that involves the notion of time in its behaviour.
- **Radar_Tx_DBS_Measurement** this model is used to calculate the radiation pattern of the transmitter rectangle array antenna. Theta or phi is linear swept to calculate the corresponding square of voltages to show the radiation pattern with theta or phi.

$$Output(k) = \left(abs \left(\sum_{i=1}^{PRI*Sampling\ Rate} \sum_{j=1}^{Num\ of\ Antx*Num\ of\ Anty} [[input(j)(i) * Rotation(j)(k))]]^2 * \exp(SweepStart + k * SweepStep)\right.\right.$$

Description:-

This block diagram shows how the switched beam is generated and target is being detected. A LFM waveform generator is used which is given to as an input to Radar_DBS_Tx which will synthesize the main lobe along the direction of theta parameter is set as number of element used sixteen and width between antenna is 0.5, this block diagram have used three Tx_DBS having three different theta values to generate a switched beam which is shown in (4.2 result) Fig. 3. A Mathlang block is used where addition of the three Tx_DBS is processed and the output is given to the Radar_MultiCH_Tx which is processing the input from baseband to RF used a 79 GHz carrier frequency, the RF output is passed through Radar_Tx_DBS_Mesurement where θ is linearly swept to calculate the corresponding square of voltages to show the radiation pattern with θ by adding a sink to the output of Radar_Tx_DBS_Mesurement. The RF output from the Radar_MultiCH_Tx is passed through two Radar_TargetEcho where the RCS type is set to automobile, again a mathlang block is used to adding the two output data from each Radar_TargetEcho. Then it is again processed through Radar_MultiCH_Rx which does the down conversion of incoming RF signals, the output is passed through an Mathlang block where the processing is done with the help of the program. After digital signal processing, the detection of two targets is achieved where the nearer target is situated at 100 M and farther target is positioned at 200 M as shown in Fig.6.

RESULTS

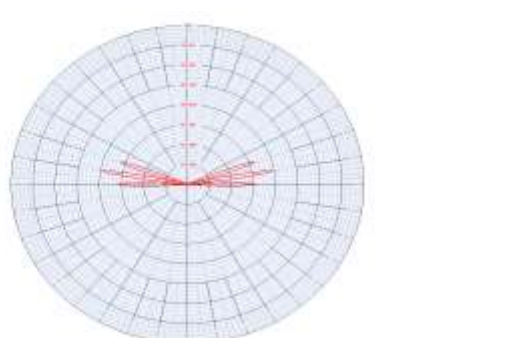

Fig. 5 Switch Beam is generated

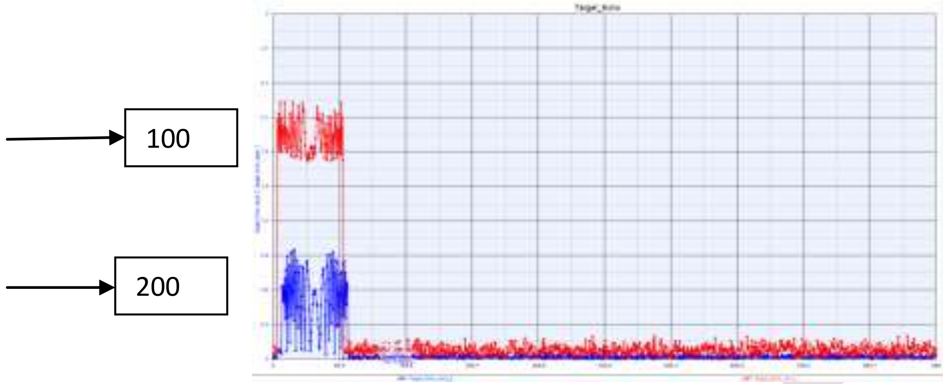

Fig. 6 Target is being Detected

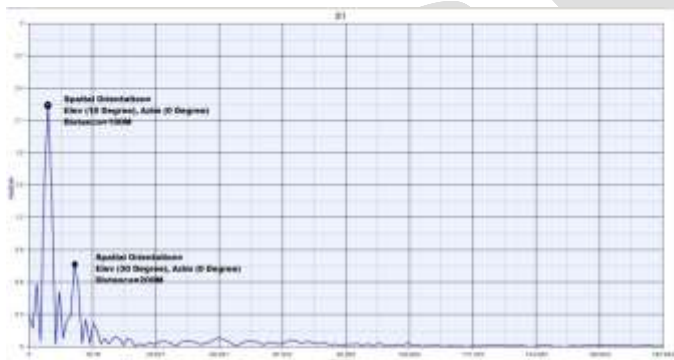

Fig. 7 Output of Pulse Compression

CONCLUSION

The paper includes the latest technology on vehicular radar having ***Target detection*** mode. Radar is with MULTI-BEAM AND MULTI-RANGE categories. On road situation the vehicular radar is able to detect multiple targets with broad antenna beam which is simulated using the concept of switching beams of Antenna array (i.e., MVDR based target detection). Multiple beams are formed at the radar transmitter using multiple DBF functions which are added together and further passed to the Uniform Rectangular Array (URA) through up-conversion blocks. Two Road vehicles are simulated at the channels and received signals are passed through MVDR processing for vehicles detection and both the vehicles are successfully detected.

Further this may be upgraded with a Target Tracking mode.

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