General network with four nodes and four activities with triangular fuzzy number as activity times

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Abstract— In many projects we have to use human judgment for determining the duration of the activities which may vary from person to person. Hence there is vagueness about the time duration for activities in network planning. Fuzzy sets can handle such vague or imprecise concepts and has application to such network. The vague activity times can be represented by triangular fuzzy numbers. In this paper a general network with fuzzy activity times is considered and conditions for critical path are obtained also we compute total float time of each activity. Several numerical examples are discussed.

Keywords- PERT, CPM, Triangular fuzzy numbers, Fuzzy activity times.

INTRODUCTION

Fuzzy numbers were first introduced by Zadeh in 1965. There after theory of fuzzy numbers was further studied and developed by Dubois and Prade, R.Yager, Mizomoto, J. Buckly and many others. Fuzzy numbers plays an important role in many applications.

The fuzzy numbers are used to represent uncertain and incomplete information in decision making, linguistic controllers, expert systems etc.But the main hurdle in the development of applications is the computational complexity. Particularly arithmetic operations on fuzzy numbers are not an easy task. Hence more attention is needed to simplify arithmetic computation with fuzzy numbers. By restricting fuzzy numbers to triangular fuzzy numbers addition and subtraction becomes simpler but still the operation of multiplication, division and max-min remains a complex processes. Therefore, some approximate methods are needed to simplify these operations.

In this paper we have obtained one such triangular approximation for multiplication which satisfies some criteria for approximation.

PRELIMINARIES

A. Definitions

A fuzzy subset *A* of a set *X* is a function $A: X \to I$, where *I* = [0, 1]. $A(x)$, is called membership of x in *A*. The set $\{x \in X \mid A(x) \geq \alpha\}$ is called α -level cut or in short α -cut of A and is denoted by A_α. The strict α -level cut of A is the set $A_{\alpha+} = \{x \in X \mid A(x) > \alpha\}$. Support of *A* is the set $A_{0+} = \{x \in X \mid A(x) > 0\}$.

If $A(x) = 1$ for some $x \in X$ then *A* is called normal fuzzy set. If each α-cut of *A* is convex then the fuzzy set *A* is called convex fuzzy set. Core of fuzzy set A is a set $A_1 = \{x \in X \mid A(x) = 1\}$. we assume X = ℝ. A fuzzy number *A* is a fuzzy subset of the set of real numbers ℝ with membership function $A: \mathbb{R} \to [0,1]$ such that *A* is normal, convex, upper semi-continuous with bounded support. If left hand curve and right hand curve are straight lines then the fuzzy number *A* is called trapezoidal fuzzy number. If the core is singleton set then the trapezoidal fuzzy number is called triangular fuzzy number.

If A is a triangular fuzzy number then it can be represented by A = (a_1, a_2, a_3). The membership function of this fuzzy number is given by,

$$A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{when } a_1 < x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{when } a_2 \le x < a_3 \\ 0 & \text{otherwise} \end{cases}$$

In this paper we use different type of representation for triangular fuzzy numbers called as ε-δ fuzzy number. The membership function of ε-δ fuzzy number is of the form,

$$r(x) = \begin{cases} \dfrac{x-(r-\varepsilon)}{\varepsilon} & \text{when } r-\varepsilon < x \le r \\ \dfrac{(r+\varepsilon)-x}{\varepsilon} & \text{when } r < x \le r+\varepsilon \\ 0 & \text{otherwise} \end{cases}$$

We denote the above triangular fuzzy number by $r_{\varepsilon,\delta}$ where ε and δ are left and right spreads of the fuzzy number. To obtain arithmetic computations with fuzzy numbers in simpler way we use this notation.

Note that A= (a_1, a_2, a_3)

is equivalent to $A = (a_2)_{a_2-a_1, a_3-a_2}$. Conversely if A= $r_{\varepsilon,\delta}$ then A= (r-ε, r, r + δ)

B. Arithmetic operations on ε-δ Fuzzy Numbers

1) Addition

Addition of ε-δ fuzzy numbers $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ is $(r+s)_{\varepsilon_1+\varepsilon_2,\ \delta_1+\delta_2}$ -fuzzy number defined by $(r_{\varepsilon_1,\delta_1} + s_{\varepsilon_2,\delta_2}) = (r+s)_{\varepsilon_1+\varepsilon_2,\delta_1+\delta_2}$.

2) Negation

Negation of ε-δ fuzzy numbers $r_{\varepsilon,\delta}$ is $-(r_{\varepsilon,\delta}) = (-r)_{\delta,\varepsilon}$.

3) Subtraction

Subtraction of ε-δ fuzzy numbers $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ is $(r-s)_{\varepsilon_1+\delta_2,\varepsilon_2+\delta_1}$ -fuzzy number defined by $(r_{\varepsilon_1,\delta_1} - s_{\varepsilon_2,\delta_2}) = (r-s)_{\varepsilon_1+\delta_2,\varepsilon_2+\delta_1}$.

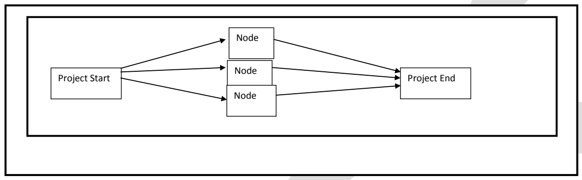
4) Maximum and minimum of fuzzy numbers

We use the max min operations introduced by Dubois-Prade [9]. If $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ are two fuzzy numbers then

Max ($r_{\varepsilon_1,\delta_1}$, $s_{\varepsilon_2,\delta_2}$) = $r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = (r \vee s)_{\varepsilon_1 \wedge \varepsilon_2, \delta_1 \vee \delta_2}$

Min ($r_{\varepsilon_1,\delta_1}$, $s_{\varepsilon_2,\delta_2}$) = $r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = (r \wedge s)_{\varepsilon_1 \vee \varepsilon_2, \delta_1 \wedge \delta_2}$

A project network is a graph (Flow chart) depicting the sequence in which a project's terminal elements are to be completed by showing terminal elements and their dependencies. The terminal elements of the project are called nodes. Nodes are connected by arcs or lines. Time required to complete the activity is written on arcs. The project network is always drawn from left to right to reflect its project's chronology.



The most popular form of project network is activity on node the other is activity on arc. PERT are CPM are the main project network techniques to identify critical path of the Network. CPM provides graphical view of the project and predicts the time required to complete the project. Network in which the activity time is fuzzy is called as fuzzy project network.

NOTATIOS USED

A network G =< V ; A >, being a project activity-on-arc model with n nodes and m activities, is given. V is the set of nodes (events) and A is the set of arcs (activities), $|V| = m$ an $|A| = n$. The network G is a directed, connected and acyclic graph. The set V = {1, 2,…, n} is labeled in such a way that i < j for each activity $(i, j) \in A$. Weights of the arcs (activity durations) $(i, j) \in A$ are to be denoted by t_{ij} .Two nodes 1 and n are distinguished as the initial and final node, respectively.

Let i-j be the activity in which 'i' is called tail event (node) and 'j' is called the head event (node). Let t_{ij} be the activity duration of an activity i-j. The earliest starting time for an activity i-j is the earliest time by which activity can commence and is denoted by ES= max ($T_E^{\,i}$ + t_{ij}) ; i $\neq$ j. Earliest finish time is the time by which it can be finished and is denoted by EF

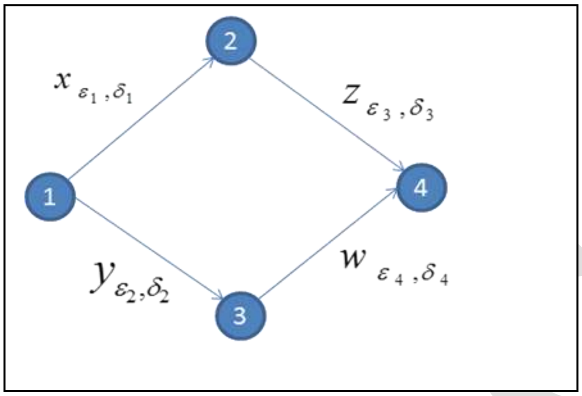
Latest starting time of an activity is the latest time by which an activity can be started without delaying the completion of the project and it is denoted by LS. Latest finish time for an activity is the time by which an activity can be finished and it is given by LF = Latest finish time = min ($T_L^{\,j} - t_{ij}$) if i $\neq$ j.

Total float (slack) at each event is given by

TF = LF – EF. Similarly Free float (FF) is FF = $F_F = F_T - S_j$ where $S_j = T_L^{\,j} - T_E^{\,j}$, and

Independent Float is IF = $F_F - S_i$,where $S_i = T_L^{\,i} - T_E^{\,i}$.

GENERAL EXAMPLE



Then earliest start time for different nodes is

$$ES_1 = 0_{0,0}\,,\ ES_2 = ES_1 + t_{12} = x_{\varepsilon_1,\delta_1}\,,\ ES_3 = ES_1 + t_{13} = 0_{0,0} + y_{\varepsilon_2,\delta_2} = y_{\varepsilon_2,\delta_2}$$

$$ES_4 = (ES_2 + t_{24}) \vee (ES_3 + t_{34}) = (x_{\varepsilon_1,\delta_1} + z_{\varepsilon_3,\delta_3}) \vee (y_{\varepsilon_2,\delta_2} + w_{\varepsilon_4,\delta_4})$$

$(x+z)_{\varepsilon_1+\varepsilon_3,\delta_1+\delta_3} \vee (y+w)_{\varepsilon_2+\varepsilon_4,\delta_2,\delta_4}$

$= (x+z) \vee (y+w)_{(\varepsilon_1+\varepsilon_3)\wedge(\varepsilon_2+\varepsilon_4),(\delta_1+\delta_3)\vee(\delta_2+\delta_4)}$

Let $(\varepsilon_1+\varepsilon_3)\wedge(\varepsilon_2+\varepsilon_4) = e\ (\delta_1+\delta_3)\vee(\delta_2+\delta_4) = d\ ES_4 = (x+z)\vee(y+w)_{e,d}$

The Latest finish time is given by $LF_4 = ES_4 = (x+z)\vee(y+w)_{e,d}\ LF_3 = LF_4 - t_{34} = (x+z) \vee (y+w)_{e,d} - w_{\varepsilon_4,\delta_4} =$

$$LF_3 = LF_4 - t_{34} = (x+z) \vee (y+w)_{e,d} - w_{\varepsilon_4,\delta_4} = ((x+z)\vee(y+w)-w)_{e+\delta_4,d+\varepsilon_4}\,,\ LF_2 = LF_4 - t_{24}$$

$= ((x+z)\vee(y+w))_{e,d} - z_{\varepsilon_3,\delta_3}$

$= ((x+z)\vee(y+w)-z)_{e+\delta_3,d+\varepsilon_3}$

$$LF_1 = (LF_2 - t_{12}) \wedge (LF_3 - t_{13})$$

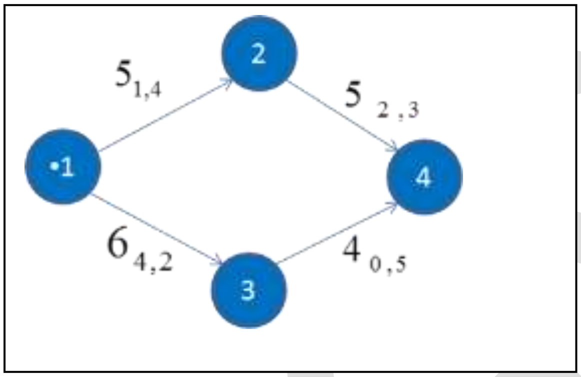
$[((x+z)\vee(y+w)-z)_{e+\delta_3,d+\varepsilon_3} - x_{\varepsilon_1,\delta_1}]\vee[((x+z)\vee(y+w)-w)_{e+\delta_4,d+\varepsilon_4} - y_{\varepsilon_2,\delta_2}]$

$[(x+z)\vee(y+w)-z-x]_{e+\delta_3+\delta_1,d+\varepsilon_3+\varepsilon_1}\vee[(x+z)\vee(y+w)-w-y]_{e+\delta_4+\delta_2,d+\varepsilon_4+\varepsilon_2}$ If $x+z > y+w$ then 1-2-4 is critical path and if $x+z < y+w$ then 1-3-4 is critical path. If $x+z = y+w$ then we can determine the critical path using following procedure. We have $LF_1 = 0_{(e+\delta_3+\delta_1)\wedge(e+\varepsilon_4+\varepsilon_2),(d+\varepsilon_3+\varepsilon_1)\vee(d+\varepsilon_4+\varepsilon_2)}$, Float on node (3) $TF_3 = LF_3 - ES_3$

$=[(x+z)\vee(y+w)-w]_{e+\delta_4,d+\varepsilon_4}-y_{\varepsilon_2,\delta_2} \quad =0_{e+\delta_4+\delta_2,d+\varepsilon_4+\varepsilon_2}$

Float on node (2) $TF_2 = LF_2 - ES_2 = [(x+z)\vee(y+w)-z]_{e+\delta_3,d+\varepsilon_3} - x_{\varepsilon_1,\delta_1} = 0_{e+\delta_3+\delta_1,d+\varepsilon_3+\varepsilon_1}$ Floats are given in the table 1

A.NUMERICAL EXAMPLE 1.

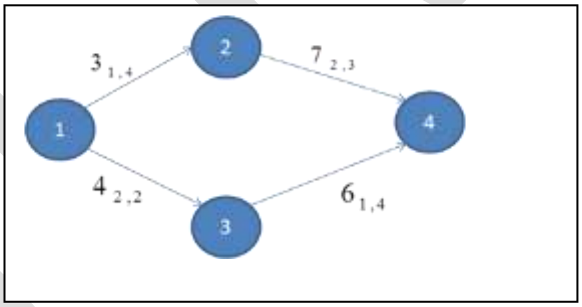


From Fig. 2 float at node (2) is, $TF_2 = LF_2 - ES_2 = 5_{6,9} - 5_{1,4} = 0_{10,10}$.

Float at node (3) is $TF_3 = LF_3 - ES_3 = 6_{8,7} - 6_{4,2} = 0_{10,11}$.The possible paths are 1-2-4 and 1-3-4,

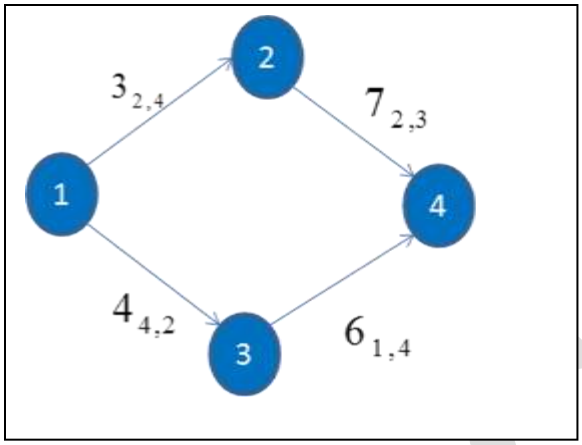
Decision nodes are (2) and (3). The floats have equal core. Therefore we compare their spreads. The left spreads are equal but right spread of (3) is greater. Hence the critical path is 1-3-4.

NUMERICAL EXAMPLE 2



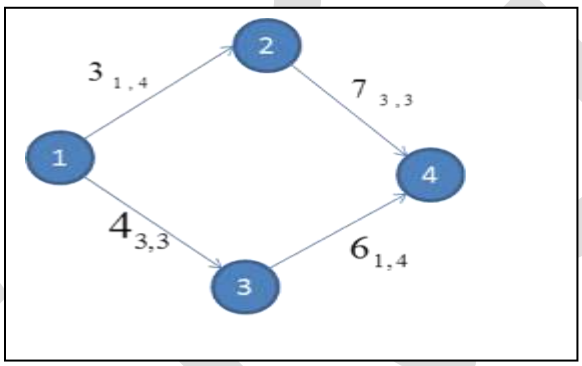
From Fig.(3) Floats at node (2) and (3) are $0_{10,10}$ and $0_{9,10}$. Here right spreads are equal therefore the node which has lesser left spread is critical. The critical path is 1-3-4.

NUMERICAL EXAMPLE 3



From fig.4 floats at nodes (2) and (3) are $0_{11,11}$ $0_{10,12}$ therefore node which has lesser left spread and greater right spread is critical. The critical path is 1-3-4.

NUMERICAL EXAMPLE 4



From fig. 5 floats at node (3) and (2) are $0_{11,11}$ Floats are equal. Therefore both the paths are critical paths and in this case there are multiple critical paths.

CONCLUSION

In this paper triangular fuzzy numbers are renamed as ε-δ fuzzy numbers are used to represent time duration of activities. A general network with four nodes and four activities is discussed. Computation of critical path is discussed with various examples.

Rules for determining nodes on the critical path are given.

NODE	Total Float	Free Float
1	$[(x+z)\vee(y+w)-x-z]_{(e+\delta_3+\delta_1),(d+\varepsilon_3+\varepsilon_1)}$	$0_{e+d+\varepsilon_1+\varepsilon_3+\delta_1+\delta_3,e+d+\varepsilon_1+\varepsilon_3+\delta_1+\delta_3}$

Table 1

2	$[(x+z)\vee(y+w)-y-w]_{(e+\delta_4+\delta_2),(d+\varepsilon_4+\varepsilon_2)}$	$0_{e+d+\varepsilon_2+\varepsilon_4+\delta_2+\delta_4,e+d+\varepsilon_2+\varepsilon_4+\delta_2+\delta_4}$
3	$[(x+z)\vee(y+w)-x-z]_{(e+\delta_1+\delta_3),(d+\varepsilon_1+\varepsilon_3)}$	$0_{2e+d+\delta_1+\delta_3,e+2d+\varepsilon_1+\varepsilon_3}$
4	$[(x+z)\vee(y+w)-y-w]_{(e+\delta_4+\delta_2),(d+\varepsilon_4+\varepsilon_2)}$	$0_{2e+d+\delta_2+\delta_4,e+2d+\varepsilon_2+\varepsilon_4}$

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