Time Truncated Chain Sampling Plan for Welbull Distributions

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Abstract: Chain sampling plan is developed for a truncated life test when the life time of an item follows Weibull distribution are provided in this manuscript. The design parameters such as the minimum sample size and the acceptance number are obtained by satisfying the consumer's risks at the specified quality levels, under the assumption that the termination time and the number of items are pre-fixed, the results are discussed with the help of tables and examples

Keywords - Truncated life test,Weibull distribution, Chain sampling plan, Operating characteristics, Consumer's risk, Producer's risk .

1. INTRODUCTION:

Sampling plans often used to determine the acceptability of lots of items. Although in recent years more emphasis is placed on process control and off-line quality control methods, acceptance sampling remains as a major tool of many practical quality control system. In acceptance sampling, if the quality variable is the lifetime of an item, the problem of acceptance sampling is known as the reliability sampling, and the test is called the lifetest. Acceptance sampling has been one of practical tools for quality assurance applications, which provide a general rule to the producer and the consumer for product acceptance determination. It has been shown that variables sampling plans requires less sampling compared with attributes sampling plans. Thus, variables sampling plans become more attractive and desirable especially when the required quality level is very high or the allowable fraction non-conforming is very small.

One can use the truncated life test, where the test can be performed without waiting until all test items fail, that reduces the test time and money significantly. In most acceptance sampling plans for a truncated life test, major issue is to determine the sample size from the lot under consideration. The problem considered here is that of finding the minimum sample size, when the life test is terminated at a preassigned T, where the number of failures is recorded until the pre specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Using life tests one may find the probability of acceptance, minimum sample size put on test and the minimum ratio of true average life to the specified average life or quality level subject to the consumer's risk. These life tests are studied by many authors using the different statistical distributions, More recently, Aslam and Jun [1] proposed the group acceptance sampling plan based on the truncated life test when the lifetime of an item follows the inverse Rayleigh and Log-logistic distribution, Kantam R.R. L., Rosaiah K. and Srinivasa Rao G. [12] discussed acceptance sampling based on life tests with Log-logistic models. Rosaiah K. and Kantam R.R.L. [13] discussed acceptance sampling based on the Inverse Rayleigh distribution. Gupta and Groll(1961), Baklizi and EI Masri(2004), and Tsai, Tzong and Shou(2006), Balakrishnan, Victor Leiva & Lopez (2007),[5]. All these authors developed the sampling plans for life tests using single acceptance sampling.

In this paper a new approach of designing Chain sampling plan for truncated life test is proposed assuming that the experiment is truncated at preassigned time ,when the lifetime of the items follows,Weibull distribution .The test termination time and the mean ratio's are specified. The design parameter is obtained such that it satisfies the consumer's risk. The probability of acceptance are also determined when the life time of the items follows the above distribution .The tables of the design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples.

2. GLOSSARY OF SYMBOLS:

n - Sample size

λ - Shape parameter

σ - Scale parameter

α - Producer's risk

T - Prefixed time

β - Consumer's risk

d - Number of defectives

p_0 - Failure probability

L(p) - Probability of acceptance

i - Acceptance criteria

3. WEIBULL DISTRIBUTION : The cumulative distribution function of the Weibull distribution is given by

$$F(t/\sigma) = 1 - e^{-\left(\frac{t}{\sigma}\right)^{\lambda}} \quad \text{------------------------} \quad \textbf{(1)}$$

Where shape parameter $\lambda = 2$, σ is the scale parameter. If some other parameters are involved, then they are assumed to be known, for example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/σ.

4. CHAIN SAMPLING PLAN : Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the shortcomings of the Single Sampling Plan The distinguish feature is that the current lot under inspection can also be accepted if one defective unit is observed in the sample provided that no other defective units were found in the samples from the immediately preceeding i lots, i.e. the chain. It avoids rejection of a lot on the basis of a single nonconfirming unit and improves the poor discrimination between good and bad quality. When large samples are not practicable, and the use of c = 0 plan is warrented, for example, when an extremely high quality is essential the use of chain sampling plan is often recommended The conditions for application and operating procedure of chsp-1 are as follows

4.1 CONDITIONS FOR APPLICATION OF ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.

2) Normally lots are expected to be of essentially the same quality.

3) The consumer has faith in the integrity of the producer..

4.2 OPERATING PROCEDURE OF CHAIN SAMPLING PLAN The plan is implemented in the following way: 1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements. 2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1. 3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n.

4.3 OPERATING PROCEDURE OF CHAIN SAMPLING PLAN FOR THE LIFE TESTS

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements during the time t_0 .

2) Accept the lot if d (the observed number of defectives during the time t_0) is zero in the sample of n units, and reject if d > 1.

3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n during the time t_0.

The Chain sampling Plan is characterized by the parameters n and i. We are interested in determining the sample size required for in the case of Weibull distribution and various values of test termination ratios. The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β.

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. The probability of acceptance in the case of chain sampling plan is calculated using the following equation is given by

$$P_a(p) = (1-p)^{n} + np(1-p)^{n-1}(1-p)^{ni} \quad \text{----------------------} \quad (2)$$

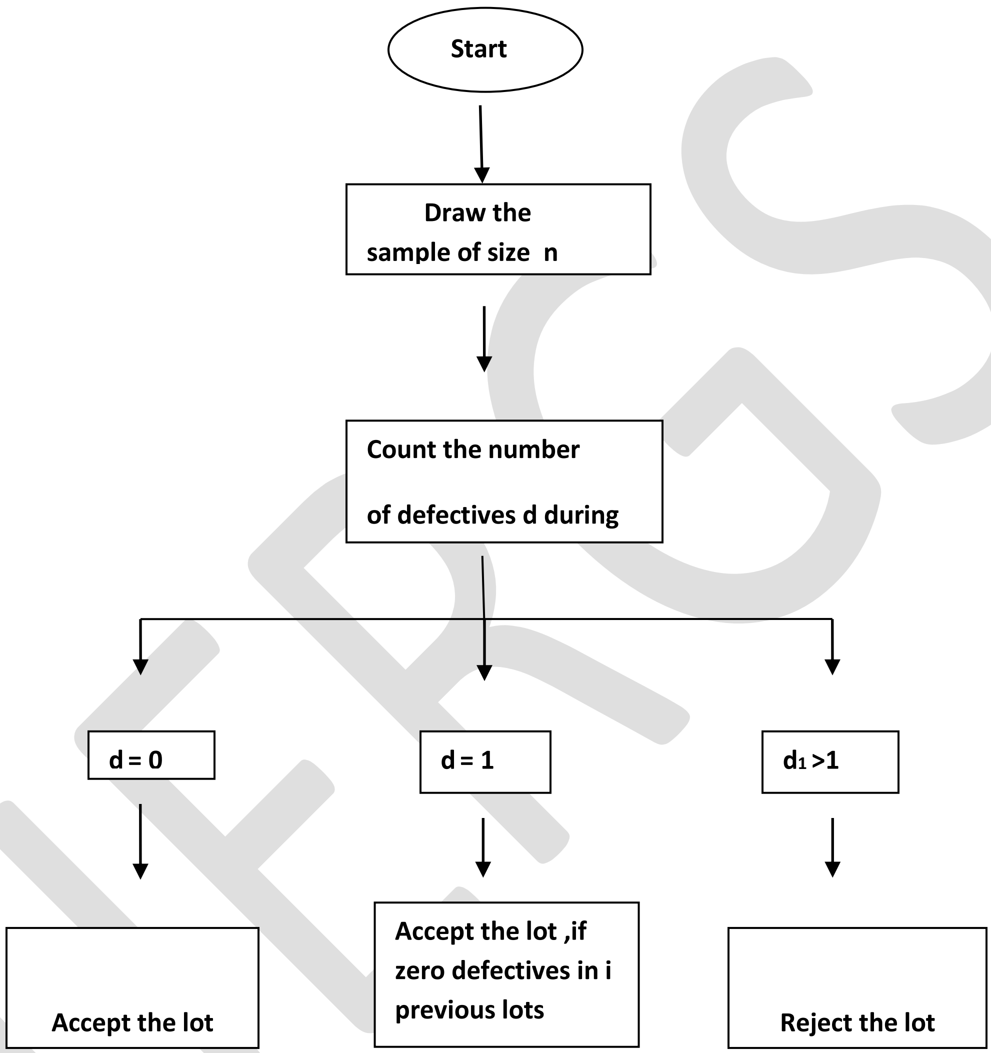
The time termination ratio t/σ_0 are fixed as 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 4.712, the consumer's risk β as 0.25, 0.10, 0.05, 0.01 and the mean ratios σ/σ_0 are fixed as 2, 4, 6, 8, 10 and 12. These choices are consistent with Gupta and Groll (1961), Gupta (1962), Kantam et al (2001), Baklizi and EI Masri (2004), Balakrishnan et Al (2007). For various time termination ratios the design parameter values n are obtained by substituting the failure probability at the worst case in the equation (2) using the inequality

$$L(p_0) \le \beta$$

where p_0 is the failure probability at $\sigma = \sigma_0$ and are presented in Table 1.The probability of acceptance for Chain sampling plan are also calculated for various time termination ratios and mean ratios and are presented in Table 2 when the life time of an item follows Weibull distribution.

5.FLOWCHART:

The following is the operating procedure of Chain sampling plan for the life tests in the form of a flow chart

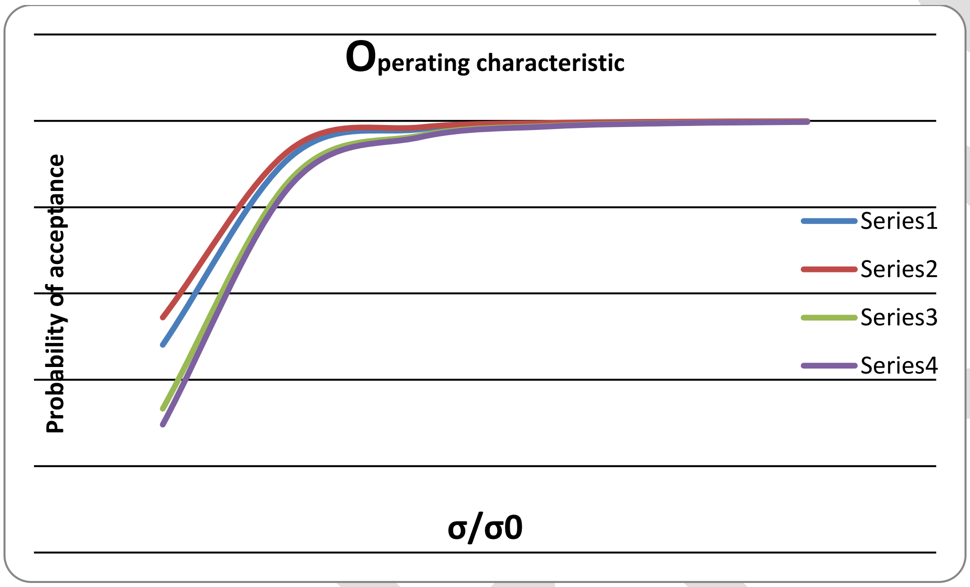


6.DESCRIPTION OF TABLES AND AN EXAMPLE:

Assume that the life time distribution is an Weibull distribution when the acceptance criteria (previous lots number) is predefined as i = 2, and that the experimenter is interested in knowing that the true mean life is atleast 1000 hours with confidence 075. Based on consumer's risk values and the time termination ratio, the minimum sample size is determined using the chain sampling plan for truncated life test. It is desired to stop the experiment at 628 hours with β= 0.25., then the required *n* from Table 1 is 4. If during 628 hours no failures out of 4 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours,and,accept the lot, if one non conforming unit is observerd, provided that no failure occurs in the i preceeding samples, and otherwise reject the lot. From the Table 2, one can observe that the probability of acceptance for this sampling is 0.801056, when $\sigma/\sigma_0 = 2$. For the same measurements and plan parameters, the probability of acceptance is 0.999720, when the ratio of the unknown average life is 12. For this sampling plan with (n = 4, i= 2, $t/\sigma_0 = 0.628$) and $\beta = 0.25$ under Weibull distribution, the values of the operating characteristic function from Table 2 as follows

σ/σ_0	**2**	**4**	**6**	**8**	**10**	**12**
L(p)	0.801056	0.980370	0.995759	0.998615	0.999424	0.999720

FIGURE 4:



OC curve for Probability of acceptance against σ/σ0 for the Chain Sampling Plan when the life time of the item follows Weibull distribution.

Table 1: Minimum sample size (n) for Chain sampling plan when the life time of the item follows Weibull distribution

β	i	t/σ_0							
		0.628	**0.942**	**1.257**	**1.571**	**2.356**	**3.141**	**3.927**	**4.712**
0.25	**1**	5	5	2	2	1	1	1	1
	2	4	2	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1
	4	4	2	1	1	1	1	1	1
	5	4	2	1	1	1	1	1	1
	6	4	2	1	1	1	1	1	1
0.10	**1**	7	6	3	2	1	1	1	1
	2	6	3	2	1	1	1	1	1
	3	6	3	2	1	1	1	1	1
	4	6	3	2	1	1	1	1	1
	5	6	3	2	1	1	1	1	1
	6	6	3	2	1	1	1	1	1
0.05	**1**	8	6	3	2	1	1	1	1
	2	8	4	2	2	1	1	1	1
	3	8	4	2	2	1	1	1	1
	4	8	4	2	2	1	1	1	1
	5	8	4	2	2	1	1	1	1
	6	8	4	2	2	1	1	1	1
0.01	**1**	12	6	3	2	1	1	1	1
	2	12	6	3	2	1	1	1	1
	3	12	6	3	2	1	1	1	1
	4	12	6	3	2	1	1	1	1
	5	12	6	3	2	1	1	1	1
	6	12	6	3	2	1	1	1	1

Table 2: **Probability of acceptance for Chain sampling plan with i = 2, when the life time of the item follows Weibull distribution**

β	n	t/σ_0	σ/σ_0					
			2	4	6	8	10	12
0.25	4	0.628	0.801056	0.980370	0.995759	0.998615	0.999424	0.999720
	2	0.942	0.772910	0.976780	0.994946	0.998345	0.99931125	0.999665
	1	1.257	0.821771	0.983147	0.996391	0.998825	0.999512	0.999763
	1	1.571	0.673600	0.962055	0.991510	0.997192	0.998826	0.999427
	1	2.356	0.296419	0.853329	0.962083	0.986770	0.994327	0.997194
	1	3.141	0.091478	0.673855	0.898854	0.962098	0.983177	0.991521
	1	3.927	0.021605	0.471422	0.799494	0.918118	0.962072	0.980420
	1	4.712	0.003900	0.296419	0.673770	0.853329	0.928605	0.962083
0.10	6	0.628	0.658855	0.958601	0.990647	0.996895	0.998700	0.999365
	3	0.942	0.615192	0.950579	0.988692	0.996229	0.998417	0.999226
	2	1.257	0.544392	0.935557	0.984922	0.994931	0.997864	0.998954
	1	1.571	0.673600	0.962055	0.991510	0.997192	0.998826	0.999427
	1	2.356	0.296419	0.853329	0.962083	0.986770	0.994327	0.997194
	1	3.141	0.091478	0.673855	0.898854	0.962098	0.983177	0.991521
	1	3.927	0.021605	0.471422	0.799494	0.918118	0.962072	0.980420
	1	4.712	0.003900	0.296419	0.673770	0.853329	0.928605	0.962083
0.05	8	0.628	0.532183	0.931526	0.983839	0.994549	0.997700	0.998873
	4	0.942	0.481087	0.918291	0.980370	0.993336	0.997179	0.998615
	2	1.257	0.544392	0.935557	0.984922	0.994931	0.997864	0.998954
	2	1.571	0.333230	0.866747	0.965932	0.988163	0.994934	0.997498
	1	2.356	0.296419	0.853329	0.962083	0.986770	0.994327	0.997194
	1	3.141	0.091478	0.673855	0.898854	0.962098	0.983177	0.991521
	1	3.927	0.021605	0.471422	0.799494	0.918118	0.962072	0.980420
	1	4.712	0.003900	0.296419	0.673770	0.853329	0.928605	0.962083

0.01	**12**	**0.628**	0.342049	0.867246	0.965918	0.988139	0.994919	0.997489
	6	**0.942**	0.291683	0.843032	0.958601	0.985444	0.993735	0.996895
	3	**1.257**	0.347265	0.871622	0.967308	0.988658	0.995149	0.997605
	2	**1.571**	0.333230	0.866747	0.965932	0.988163	0.994934	0.997498
	1	**2.356**	0.296419	0.853329	0.962083	0.986770	0.994327	0.997194
	1	**3.141**	0.091478	0.673855	0.898854	0.962098	0.983177	0.991521
	1	**3.927**	0.021605	0.471422	0.799494	0.918118	0.962072	0.98042
	1	**4.712**	0.003900	0.296419	0.673770	0.853329	0.928605	0.962083

7.CONCLUSIONS: In this paper, designing a Chain sampling plan for the truncated life test is presented. The minimum sample size and the probability of acceptance are calculated, for various values of the test termination time, assuming that the lifetime of an item follows Weibull distribution. From the figure1 and the tables provided, it is observed that the operating characteristic values of Weibull distribution increases disproportionately and reaches the maximum value 1 with the increase in the mean ratios. It is concluded that the sampling plan can be used conveniently in practical situations to reduce the cost and time of the life test experiments.

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