Wavelet Analysis for Nanoscopic TEM Biomedical Images with Effective Weiner Filter

Garima Goyal
goyal.garima18@gmail.com
Assistant Professor, Department of Information Science & Engineering
Jyothy Institute of Technology, Bangalore

ABSTRACT- Wavelet transforms and other multi-scale analysis functions have been used for compact signal and image representations in de-noising, compression and feature detection processing problems for about twenty years. The wavelet transform itself offers great design flexibility. Basis selection, spatial-frequency tiling, and various wavelet threshold strategies can be optimized for best adaptation to a processing application, data characteristics and feature of interest. One of the most important features of wavelet transforms is their multi-resolution representation. In this paper the complete wavelet family is analysed with its in combination in best with Wiener Filter aiming to denoise at the same time. The medical Nanoscopic TEM image will be analysed with different wavelets. The filtered image is further analysed on the basis of Mean, MSE, SNR & PSNR.
Keywords: Medical Image, Wavelets, Noise, Wiener Filter, TEM, SNR, PSNR

I INTRODUCTION TO BIOMEDICAL IMAGES

One of the most fundamental problems in signal processing is to find a suitable representation of the data that will facilitate an analysis procedure. One way to achieve this goal is to use transformation, or decomposition of the signal on a set of basis functions prior to processing in the transform domain. Transform theory has played a key role in image processing for a number of years, and it continues to be a topic of interest in theoretical as well as applied work in this field. Image transforms are used widely in many image processing fields, including image enhancement, restoration, encoding, and description. [10][11]

II INTRODUCTION TO WAVELETS AND WIENER FILTER

Historically, the Fourier transform has dominated linear time-invariant signal processing. The associated basis functions are complex sinusoidal waves $e^{i\omega t}$ that correspond to the eigenvectors of a linear time-invariant operator. A signal $f(t)$ defined in the temporal domain and its Fourier transform $\hat{f}(\omega)$, defined in the frequency domain, have the following relationships.

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt, \quad (1)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{f}(\omega) e^{i\omega t} d\omega. \quad (2)$$

Fourier transform characterizes a signal $f(t)$ via its frequency components. Since the support of the bases function $e^{i\omega t}$ covers the whole temporal domain (i.e infinite support), $\hat{f}(\omega)$ depends on the values of $f(t)$ for all times. This makes the Fourier transform a global transform that cannot analyze local or transient properties of the original signal $f(t)$.

In order to capture frequency evolution of a non-static signal, the basis functions should have compact support in both time and frequency domain. To achieve this goal, a windowed Fourier transform (WFT) was first introduced with the use of a window function *w(t)* into the Fourier transform:

$$Sf(\omega, t) = \int_{-\infty}^{+\infty} f(\tau) w(t-\tau) e^{-i\omega\tau} d\tau. \quad (3)$$

The energy of the basis function $g_{\tau,\xi}(t) = w(t-\tau)e^{-i\xi t}$ is concentrated in the neighbourhood of time τ over an interval of size σ_t, measured by the standard deviation of $|g|^2$. Its Fourier transform is $\hat{g}_{\tau,\xi}(\omega) = \hat{w}(\omega-\xi)e^{-i\tau(\omega-\xi)}$, with energy in frequency domain localized around ξ, over an interval of size σ_ω. In a time-frequency plane (t,ω), the energy spread of what is called the atom $g_{\tau,\xi}(t)$ is represented by the Heisenberg rectangle with time width σ_t and frequency width σ_ω. The uncertainty principle states that the energy spread of a function and its Fourier transform cannot be simultaneously arbitrarily small, verifying:

$$\sigma_t \sigma_\omega \geq \frac{1}{2}. \quad (4)$$

Shape and size of Heisenberg rectangles of a windowed Fourier transform therefore determine the spatial and frequency resolution offered by such transform. [6]

Examples of spatial-frequency tiling with Heisenberg rectangles are shown in Figure 1. Notice that for a windowed Fourier transform, the shape of the time-frequency boxes are identical across the whole time-frequency plane, which means that the analysis resolution of a windowed Fourier transform remains the same across all frequency and spatial locations.

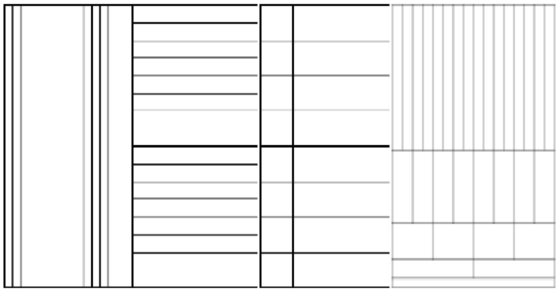


Figure 8: Example of spatial-frequency tiling of various transformations. x-axis: spatial resolution. y-axis: frequency resolution. (a) discrete sampling (no frequency localization). (b) Fourier transform (no temporal localization). (c) windowed Fourier transform (constant Heisenberg boxes). (d) wavelet transform (variable Heisenberg boxes).

To analyze transient signal structures of various supports and amplitudes in time, it is necessary to use time-frequency atoms with different support sizes for different temporal locations. For example, in the case of high frequency structures, which vary rapidly in time, we need higher temporal resolution to accurately trace the trajectory of the changes; on the other hand, for lower frequency, we will need a relatively higher absolute frequency resolution to give a better measurement on the value of frequency. We will show in the next section that wavelet transform provide a natural representation which satisfies these requirements, as illustrated in **Error! Reference source not found.** (d).[9]

III DISCRETE WAVELET TRANSFORM

Given a 1-D signal of length *N*, $\{f(n), n=0,\cdots,N-1\}$, the discrete orthogonal wavelet transform can be organized as a sequence of discrete functions according to the scale parameter $s=2^j$:

$$\left\{L_J f, \{W_j f\}_{j\in[1,J]}\right\} \qquad (5)$$

Where

$L_J f = Lf(2^J n, 2^J)$ and $W_j f = Wf(2^j n, 2^j)$.

Wavelet coefficients $W_j f$ at scale $s=2^j$ have a length of $N/2^j$ and the largest decomposition depth *J* is bounded by the signal length *N* as ($\sup(J)=\log_2 N$).

For fast implementation (such as filter bank algorithms), a pair of conjugate mirror filters (CMF) *h* and *g* can be constructed from the scaling function ϕ and wavelet function ψ as follows:

$$h[n] = <\frac{1}{\sqrt{2}}\phi(\frac{t}{2}), \phi(t-n)> \text{ and } g[n] = <\frac{1}{\sqrt{2}}\psi(\frac{t}{2}), \phi(t-n)> \qquad (6)$$

A conjugate mirror filter *k* satisfies the following relation:

$$\left|\hat{k}(\omega)\right|^2 + \left|\hat{k}(\omega+\pi)\right|^2 = 2 \text{ and } \hat{k}(0) = 2 \qquad (7)$$

It can be proven that *h* is a low-pass filter, and *g* is a high-pass filter. The discrete orthogonal wavelet decomposition in Equation (5) can be computed by applying these two filters to the input signal, and recursively decompose the low-pass band, as illustrated in **Error! Reference source not found.**.

For orthogonal basis, the input signal can be reconstructed from wavelet coefficients computed in Equation (5) using the same pair of filters, as illustrated in **Error! Reference source not found.**.

It is easy to prove that the total amount of data after a discrete wavelet expansion as shown in **Error! Reference source not found.** has the same length to the input signal. Therefore, such transform provides a compact representation of the signal suited for data compression as wavelet transform provides a better spatial-frequency localization. On the other hand, since the data was downsampled at each level of expansion, such transform performs poorly on localization or detection problems. Mathematically, the transform is variant under translation of the signal (i.e. is dependent of the downsampling scheme used during the decomposition), which makes it less attractive for analysis of non-stationary signals.

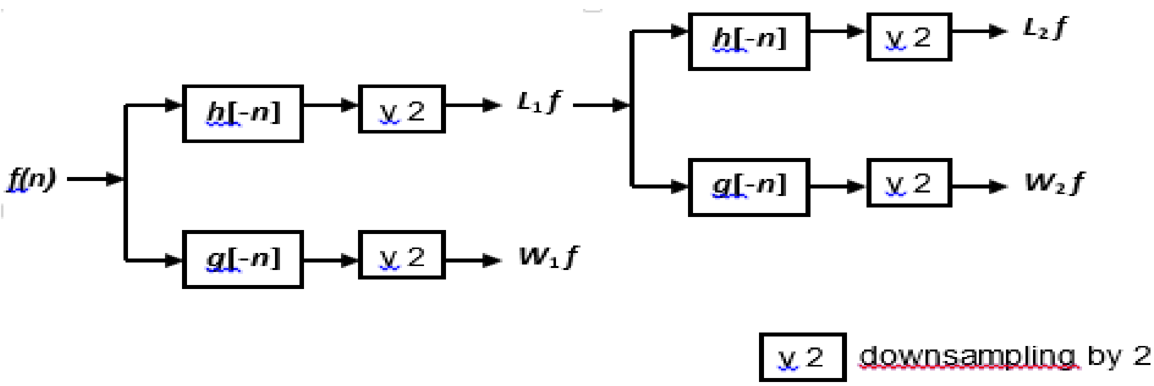


Figure 9: Illustration of orthogonal wavelet transform of a discrete signal *f(n)* with CMF. A two-level expansion is shown

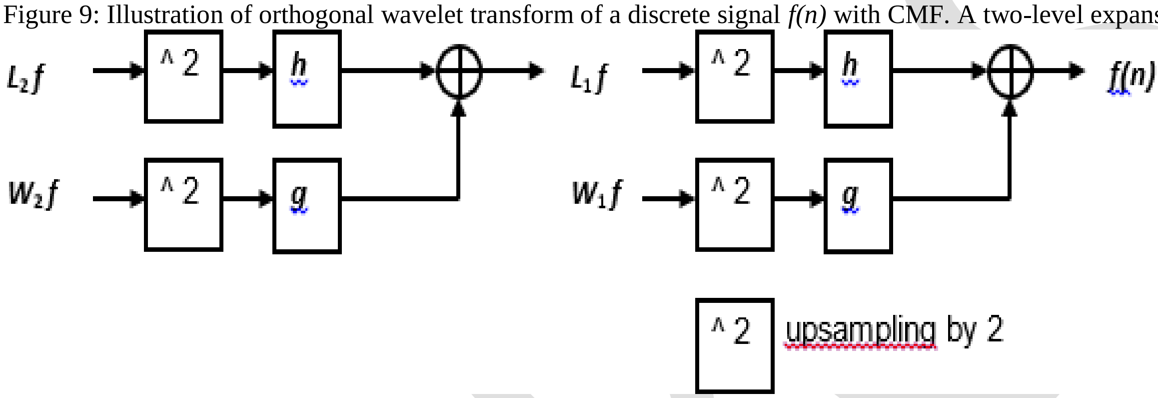


Figure 10: Illustration of inverse wavelet transform implemented with CMF. A two-level expansion is shown

In image analysis, signal and a redundant representation needs to be represented. In the dyadic wavelet transform framework proposed by Mallat and Zhong, sampling of the translation parameter was performed with the same sampling period as the input signal to preserve translation invariance.

A more general framework of wavelet transform can be designed with different reconstruction and decomposition filters that form a bi-orthogonal basis. Such generalization provides more flexibility in the design of the wavelet functions. In that case, similarly to Equation (5), the discrete dyadic wavelet transform of a signal *s(n)* is defined as a sequence of discrete functions

$$\{S_M s(n), \{W_m s(n)\}_{m\in[1,M]}\}_{n\in\mathbb{Z}}, \qquad (8)$$

where $S_M s(n) = s * \phi_M(n)$ represents the DC component, or the coarsest information from the input signal.

Given a pair of wavelet function $\psi(x)$ and reconstruction function $\chi(x)$, the discrete dyadic wavelet transform (decomposition and reconstruction) can be implemented with a fast filter bank scheme using a pair of decomposition filters *H*, *G* and a reconstruction filter *K*.

$$\begin{aligned} \hat{\phi}(2\omega) &= e^{-i\omega s} H(\omega)\hat{\phi}(\omega), \\ \hat{\psi}(2\omega) &= e^{-i\omega s} G(\omega)\hat{\psi}(\omega), \qquad (9) \\ \hat{\chi}(2\omega) &= e^{i\omega s} K(\omega)\hat{\chi}(\omega). \end{aligned}$$

where *s* is a $\psi(x)$ dependent sampling shift. The three filters satisfy:

$$|H(\omega)|^2 + G(\omega)K(\omega) = 1. \qquad (10)$$

Defining $F_s(\omega) = e^{-i\omega s} F(\omega)$, where *F* is either *H*, *G* or *K*, we can construct a filter bank implementation of the discrete dyadic wavelet transform as illustrated in **Error! Reference source not found.**. Filters $F(2^m\omega)$ defined at level *m*+1 (i.e., filters applied at wavelet scale 2^m) are constructed by inserting $2^m - 1$ zeros between subsequent filter coefficients from level 1 ($F(\omega)$). Non-integer shifts at level 1 are rounded to the nearest integer. It has a complexity that increases linearly with the number of analysis levels.[9][12]

In image processing applications, we often deal with two, three or even higher dimensional data. Multi-dimensional wavelet bases can be constructed with tensor products of separable basis functions defined along each dimension. In that context, a *N*-D discrete dyadic wavelet transform with *M* analysis levels is represented as a set of wavelet coefficients:

$$\left\{S_M s, \{W_m^1 s, W_m^2 s, \cdots, W_m^N s\}_{m=[1,M]}\right\} \quad (11)$$

11where $W_m^k s = < s, \psi_m^k >$ represents the detailed information along the *k*th coordinate at

scale *m*. The wavelet basis is dilated and translated from a set of separable wavelet functions $\psi^k, k = 1, \cdots N$ as for example in 3D:

$$\psi^k_{m,n_1,n_2,n_3}(x,y,z) = \frac{1}{2^{3m/2}} \psi^k\left(\frac{x-n_1}{2^m}, \frac{y-n_2}{2^m}, \frac{z-n_3}{2^m}\right), \; k = 1,2,3. \quad (12)$$

In this framework, reconstruction with a *N*-D dyadic wavelet transform requires a non-separable filter L_N to compensate the inter-dimension correlations. This is formulated in a general context as:

$$\sum_{l=1}^{N} K(\omega_l) G(\omega_l) L_N(\omega_1, \ldots, \omega_{l-1}, \omega_{l+1}, \ldots, \omega_N) + \prod_{l=1}^{N} |H(\omega_l)|^2 = 1 \quad (13)$$

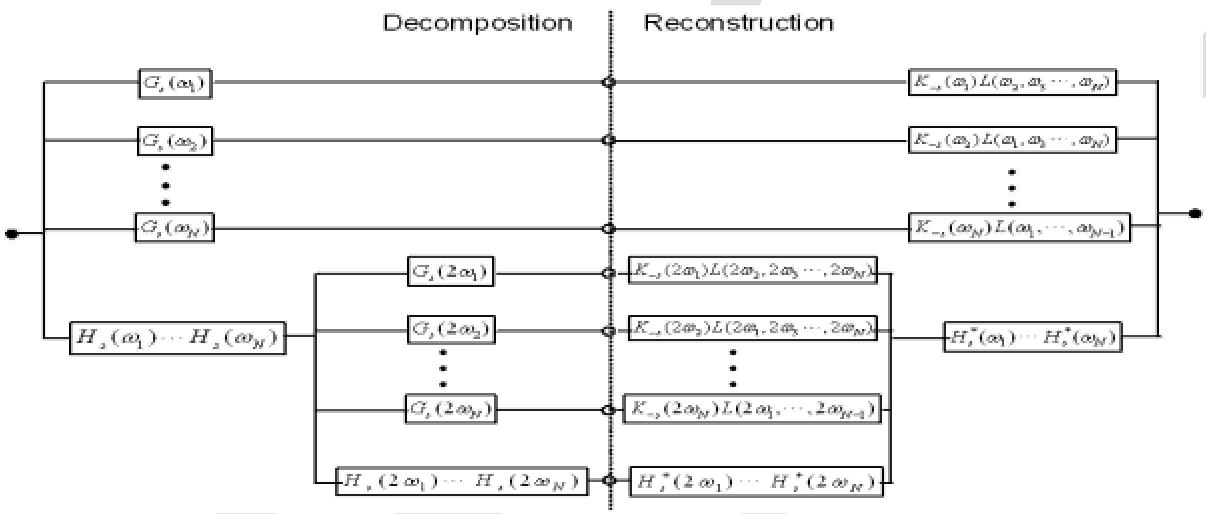
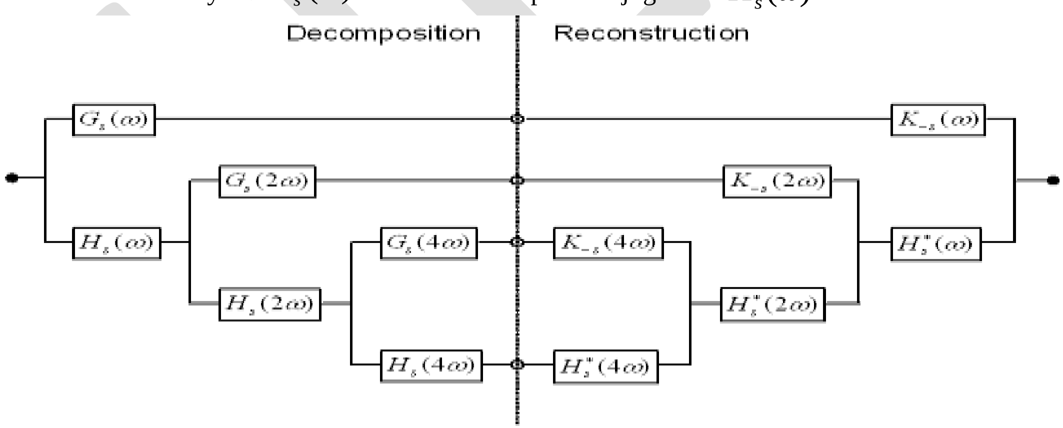


Figure 11: Filter bank implementation of a one-dimensional discrete dyadic wavelet transform decomposition and reconstruction for three levels of analysis. $H_s^*(\omega)$ denotes the complex conjugate of $H_s(\omega)$



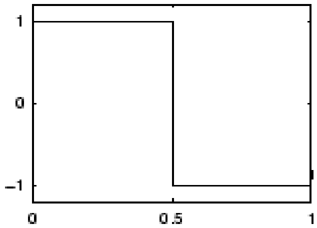
Error! Reference source not found. illustrates a filter bank implementation with a multi-dimensional discrete dyadic wavelet transform

IV WAVELET FAMILY

Wavelet families vary in terms of several important properties. Examples include:

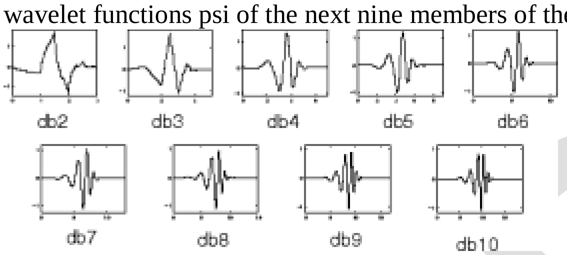
- Support of the wavelet in time and frequency and rate of decay.
- Symmetry or antisymmetry of the wavelet. The accompanying perfect reconstruction filters have linear phase.
- Number of vanishing moments. Wavelets with increasing numbers of vanishing moments result in sparse representations for a large class of signals and images.
- Regularity of the wavelet. Smoother wavelets provide sharper frequency resolution. Additionally, iterative algorithms for wavelet construction converge faster.
- Existence of a scaling function, φ.[3][7][8]

Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1.



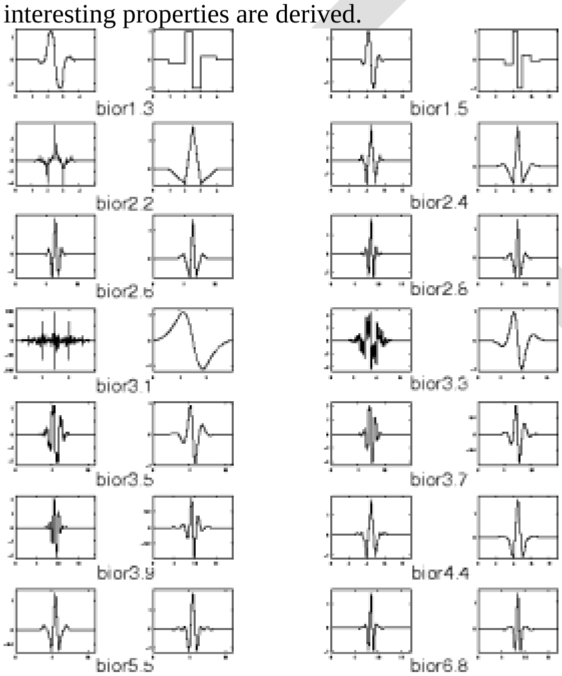
Wavelet function psi

Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the "surname" of the wavelet. The db1 wavelet, as mentioned above, is the same as Haar wavelet. Here are the wavelet functions psi of the next nine members of the family:



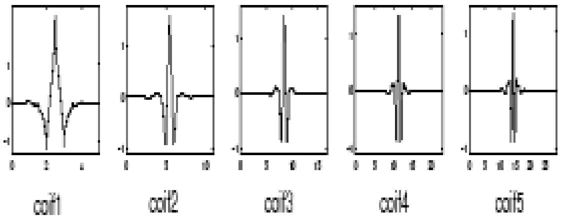
Biorthogonal

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived.

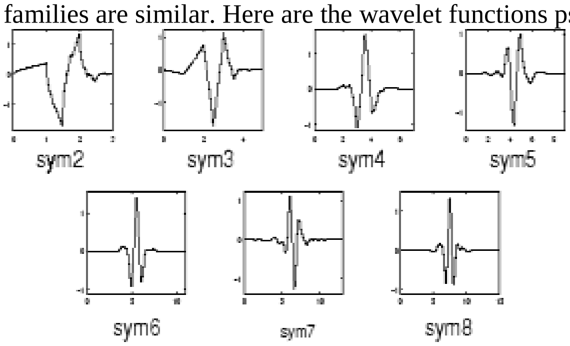


Coiflets

Built by I. Daubechies at the request of R. Coifman. The wavelet function has $2N$ moments equal to 0 and the scaling function has $2N$-1 moments equal to 0. The two functions have a support of length $6N$-1.

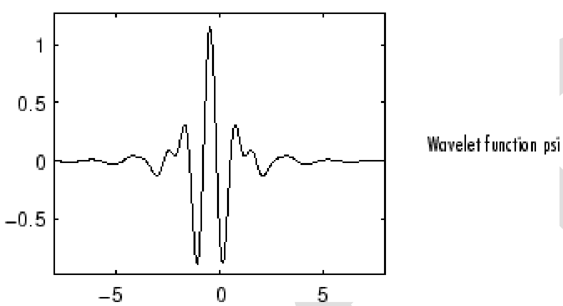


Symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Here are the wavelet functions psi.



Dmeyer

The Meyer wavelet and scaling function are defined in the frequency domain.



V PROPOSED ALGORITHM

It makes use of two level DWT. In first level DWT , a single level two dimensional wavelet decomposition and does soft thresholding by using mask filter for high frequency subband. At second level of DWT, it again do soft thresholding by further doing the single level two dimensional decomposition of on the approximation coefficient obtained in first level decomposition. It use wiener filter .i.e uses low pass filters a image that has been degraded by a constant power additive noise. It makes use of pixelwise adaptive wiener method based on the statistics estimated from a local neighborhood of each pixel. It then applies inverse 2D wavelet transform for second level decomposition. Then, applies the same for the first level decomposition. It De-noises image using Wiener filter for Low frequency domain and using new equation as a soft-thresholding for de-noise high-frequencies domains [5].This filter takes five things as input parameters namely, an input image Im, name of the wavelet family wname, mask filter used for low frequency sub-band MASKL, mask filter used for high frequency sub-band MASKH, factor >=0.001, used to decrease or increase estimated power of a noise used by wiener filter. [2][4]

Most importantly, in this paper we have analysed molecular images specially nanoscopic TEM images for different wavelet family. The wavelet family for which it has been analysed are Haar, Daubechies, Biorthogonal , Coiflets, Symlets, Reverse Biorthogonal, Discrete approximation of Meyer Wavelet.

Firstly, it computes the number of layers Layer_C, for the number of layers, it performs the following steps: it applies I level discrete wavelet transform on the input image, then II level DWT transform of the first coefficient obtained in first level transformation. Thereafter, applies soft thresholding on horizontal, vertical & diagonal approximation coefficients obtained in first level DWT. It then applies soft thresholding on horizontal, vertical and diagonal approximation coefficients obtained in second level DWT. Then maximum additive noise power is computed before applying filter. It then applies wiener filter. After applying filtering process, inverse discrete wavelet transform is applied for second level. It then checks number of columns. Thereafter inverse discrete wavelet transform is applied for first level.

VI WAVELET ANALYSIS DATASHEET

SNR							
Noise Intensity	**Noisy**	**haar**	**coif1**	**sym2**	**dmey**	**bior1.1**	**rbio1.1**
0.001	6.55	6.72	7.69	7.65	7.96	6.74	6.74
0.003	6.50	6.71	7.63	7.62	7.93	6.69	6.71
0.005	6.45	6.66	7.60	7.57	7.90	6.68	6.68
0.007	6.46	6.65	7.55	7.51	7.84	6.65	6.65
0.1	6.39	6.61	7.51	7.47	7.75	6.58	6.61
0.3	5.91	6.15	6.91	6.88	7.11	6.13	6.13
0.5	5.35	5.55	6.19	6.20	6.32	5.54	5.55

PSNR							
Noise Intensity	**Noisy**	**haar**	**coif1**	**sym2**	**dmey**	**bior1.1**	**rbio1.1**
0.001	15.5	16.6	18.3	18.2	18.9	16.6	16.6
0.003	15.4	16.6	18.2	18.1	18.9	16.5	16.6
0.005	15.3	16.5	18.1	18.0	18.8	16.5	16.5
0.007	15.3	16.5	18.1	17.9	18.7	16.5	16.5
0.1	15.2	16.4	18.0	17.9	18.6	16.4	16.4
0.3	14.4	15.7	17.0	16.9	17.5	15.7	15.7
0.5	13.5	14.8	15.8	15.8	16.1	14.7	14.8

MSE							
Noise Intensity	**Noisy**	**haar**	**coif1**	**sym2**	**Dmey**	**bior1.1**	**rbio1.1**
0.001	3.01E+07	2.33E+07	1.58E+07	1.62E+07	1.37E+07	2.32E+07	2.32E+07
0.003	3.07E+07	2.34E+07	1.61E+07	1.65E+07	1.38E+07	2.36E+07	2.35E+07
0.005	3.12E+07	2.37E+07	1.63E+07	1.66E+07	1.39E+07	2.36E+07	2.36E+07
0.007	3.10E+07	2.39E+07	1.66E+07	1.71E+07	1.42E+07	2.38E+07	2.38E+07
0.1	3.19E+07	2.41E+07	1.68E+07	1.73E+07	1.47E+07	2.44E+07	2.41E+07
0.3	3.82E+07	2.84E+07	2.12E+07	2.16E+07	1.88E+07	2.86E+07	2.86E+07
0.5	4.71E+07	3.55E+07	2.79E+07	2.81E+07	2.58E+07	3.57E+07	3.55E+07

VII CONCLUSION

The basic idea of wavelet analysis is to use a cluster of wavelet functions to express a signal. It has a high time-frequency resolution in low frequency bands, a high time resolution and low frequency resolution in high frequency bands. The decomposition sequence obtained with Fourier transform has a high time-frequency resolution and same bandwidth in the whole time-frequency domain. This indicates the special feature of the given signal. The dmey wavelet family proved better results when implemented with Fourier transform along with Wiener Filter to reduce noise at the same time enhancing the image

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