MODIFIED SELECTED MAPPING TECHNIQUE TO REDUCE PEAK TO AVERAGE POWER RATIO FOR OFDM SIGNALS

Tanvir Ahmed, Foisal Ahmed, Sifat Hossain
Department of Electrical & Electronic Engineering
Islamic University of Technology
Gazipur, Bangladesh

Abstract— High peak to average power ratio of the transmitted signal is the major limitation of the orthogonal frequency division multiplexing technique. The aim of this paper is to approach for a new modified selected mapping (SLM) technique. The modified technique has the inclusion of the idea of sub-block partitioning of signals. Then, from a comparative analysis between the conventional SLM scheme and the modified SLM scheme, it is shown that the modification gives better complementary cumulative distributive function (CCDF) of the PAPR of transmitted signal. All the simulations works are done in matlab.

Keywords— Orthogonal Frequency Division Multiplexing (OFDM), Complementary cumulative distribution function (CCDF), Peak-to-average power ratio (PAPR), Selected mapping technique (SLM), Inverse Fast Fourier Transform (IFFT), Phase Rotation, Sub-block.

INTRODUCTION

At present, high data rate and speedy communication has become the ultimate goal of any communication-research work. Multiplexing of different message signal bits into one data stream plays an important role in sending information to the receiver. There are several multiplexing techniques like time division multiplexing (TDM), frequency division multiplexing (FDM), space division multiplexing, and code division multiplexing (CDM). Third generation systems use Wide-band Code Division Multiple Access (WCDMA) as the carrier modulation scheme [1] and can achieve data rate upto 64 kbps-2Mbps [2]. But applications like high definition TV, computer network applications, online video streaming require more data rate (2-100 Mbps). Thus evolution of fourth generation technology with maximum quality of service (QOS) and optimum spectral efficiency at low cost is expected most. Orthogonal frequency division multiplexing is a potential candidate to achieve these goals and it has the potential to surpass the capacity of CDMA system. This OFDM scheme transmits data through a channel or multipath environment using several orthogonally overlapped subcarriers. Though the idea of OFDM is not a very recent issue [3-7], still it has some major drawbacks like high sensitivity to carrier offset and high peak to average power ratio (PAPR). The peak to average power ratio of the input signal plays an important role in determining the efficiency of high power amplifier (HPA). As the HPA is usually operated at or near the saturation region to achieve the maximum output power efficiency, and so the memory-less nonlinear distortion due to high PAPR of the input signals will be introduced into the communication channels. If the HPA is not operated in linear region with large power back-off, it is impossible to keep the out-of-band power below the specified limits. It results into very inefficient amplification and expensive transmitters. High PAPR problem may nullify many potential benefits of multicarrier transmission systems in case of different low-cost applications [8].

So undoubtedly, it is important and necessary to research on the characteristics of the PAPR including its distribution and reduction in OFDM systems in order to utilize the technical features of the OFDM. As one of characteristics of the PAPR, the distribution of PAPR, which bears stochastic characteristics in OFDM systems, often can be expressed in terms of Complementary Cumulative Distribution Function (CCDF). Some researches are done on determination of the PAPR distribution based on different models and hypotheses [9]–[13]. Besides, various techniques have also been proposed to reduce the PAPR [14]. Selected mapping technique is one of the promising techniques among them which affiliate the idea of scrambling signals.

In this study, firstly we discuss about the characteristics of the OFDM signals. We penetrate through the CCDF distribution of PAPR of the OFDM signals at the later part. Then, we totally lock our focus on selected mapping technique as a very convenient way for the reduction of PAPR. Finally, a new modified SLM technique is proposed and a comparative analysis is done to achieve the goal of finding a better PAPR performance.

CHARACTERISTICS OF OFDM SIGNAL

In OFDM transmission scheme, the information signals from multiple stations are combined into a single multiplexed stream of data. This data is then transmitted using an OFDM ensemble that is made up from a dense packing of many subcarriers. These multiple subcarriers overlap in the frequency domain, but do not cause Inter-Carrier Interference (ICI) due to the orthogonal nature of the modulation. The orthogonal nature of the transmission is a result of the peak of each subcarrier corresponding to the nulls of all other subcarriers.

An OFDM transmitter maps the message bits into a sequence of PSK or QAM symbols and converts it into N parallel streams. Each of N symbols from serial-to-parallel (S/P) conversion is carried out by the different subcarrier. We consider $X_l[k]$ denote the lth transmit symbol at the kth subcarrier where k=0,1,2,3…N-1. Due to the S/P conversion, the duration of transmission time for N symbols is extended to NTs, which forms a single OFDM symbol with a length of T_{sym}. If $\Psi_{l.k}(t)$ denote the lth OFDM signal at the kth subcarrier, which is given as

$$\Psi_{l.k}(t) = \{e^{j2\pi f_k (t - lT_{sym})} \quad \text{when } 0 < t < T_{sym} \qquad (1)$$

Then the passband and baseband OFDM signals in the continuous-time domain can be expressed respectively as

$$x_l(t) = \mathrm{Re}\{\frac{1}{T_{sym}} \sum_{l=0}^{\infty} \{\sum_{k=0}^{N-1} X_l[k] \Psi_{l.k}(t)\}\} \qquad (2)$$

The continuous-time baseband OFDM signal can be sampled at t= $lT_{sym} + nT_s$ with $T_s = {}^{T_{sym}}\!/_{N}$ to yield the corresponding discrete-time OFDM symbol as

$$x_l[n] = \sum_{k=0}^{N-1} X_l[k] e^{j2\pi fkn/N} \qquad \text{for n=0, 1… n-1} \qquad (3)$$

CONCEPT OF PAPR

The ratio between the maximum power and the average power of the complex passband signal $\tilde{s}(t)$ is represented by PAPR [15], that is,

$$\mathrm{PAPR}\{\tilde{s}(t)\} = \frac{\max\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2}{E\left\{\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2\right\}} = \frac{\max |s(t)|^2}{E\{|s(t)|^2\}} \qquad (4)$$

The maximum power occurs when all of the N subcarrier components are added with identical phases in case of OFDM. The maximum power will be equivalent to N times the average power that is PAPR=N for the assumption $E\{|s(t)|^2\} = 1$. Whereas, the probability of the occurrence of the maximum power signal increases as N decreases [16]. Considering M^2 OFDM signals with the maximum power among M^N OFDM signals, the occurrence probability of the largest PAPR is $M^2/M^N = M^{2-N}$, that turns out to be 4.7 x 10^{-38} in the case of OFDM with N=64. [17]. This indicates rare occasion of largest PAPR. While the input signals of N-point IFFT have the uniformly distributed independent and finite magnitudes which are for QPSK and QAM, we can assume that the real and imaginary parts have asymptotically Gaussian distributions for a sufficiently large number of subcarriers by the central limit theorem. The amplitude of the OFDM signal $s(t)$ then follows a Rayleigh distribution. Let $\{Z_n\}$ the magnitudes of complex sample $\{|s(nT_s/N|\}_{n=0}^{N-1}$ .Assuming that the average power of $s(t)$ is equal to one, that is, $E\{|s(t)|^2\} = 1$, then$\{Z_n\}$ are the independent and identically distributed Rayleigh random variables normalized with its own average power, which has the following probability density function:

$$f_{Z_n}(z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} = 2ze^{-z^2}, \; n = 0, 1, 2, \cdots, N\text{-}1 \qquad (5)$$

where $E\{Z_n^2\} = 2\sigma^2 = 1$. If Z_{max}denote the crest factor, the cumulative distribution function (CDF) of $Z_{max}$is given as:

$$F_{Z_{max}}(z) = P(Z_{max} < z)$$

$$= \left(1\text{-}e^{-z^2}\right)^N \qquad (6)$$

where $P(Z_n < z) = \int_0^2 f_{Z_n}(x)\,dx,\ \ n = 0,1,2,\cdots,N\text{-}1$

In order to find the probability that the crest factor (CF) exceeds z, considering the following complementary CDF (CCDF):

$$\tilde{F}_{Z_{max}}(z) = P(Z_{max} > z)$$

$$= 1\text{-}P(Z_{max} \le z)$$

$$= 1\text{-}F_{Z_{max}}(z)$$

$$= 1\text{-}(1\text{-}e^{-z^2})^N \qquad (7)$$

Since earlier equations are derived assuming that N samples are independent and N is sufficiently large, they do not hold for the band limited or for oversampled signals. However, deriving the exact CDF for the oversampled signal is difficult and therefore, the following simplified CDF will be used:

$$F_z(z) \approx \left(1\text{-}e^{-z^2}\right)^{\alpha N} \qquad (8)$$

where α has to be determined by fitting the theoretical CDF into the actual one. The PAPR defined earlier deals with the passband signal with a carrier frequency of $f_c$in the continuous time domain. However, the PAPR for the discrete-time baseband signal x[n]may not be the same as that for the continuous-time baseband signalx(t).For an L-times-interpolated signal, the PAPR is can be redefined as

$$PAPR = \frac{\max_{m=0,1,\ldots,NL}|x'[m]|^2}{E\{|x'[m]|^2\}} \qquad (9)$$

CONVENTIONAL SLM TECHNIQUE

Selected Mapping (SLM) technique is one of the most promising reduction techniques to reduce Peak to Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) system. PAPR reduction through SLM scheme was brainchild of Bauml, Fischer and Huber [18].The basic idea of this technique is based on the phase rotation. The lowest PAPR signal will be selected for transmission from a number of different data blocks which have independent phase sequences but same information at the transmitter. Fig. 2 shows a block diagram of SLM scheme [19]. The total selected mapping procedure can be divided into a number of steps. First of all, the transmitter generates a set of sufficiently different candidate data blocks where all of them represent the same information as the original data block; we assume input data set is $X = [X_0, X_1, X_2, X_3, ...X_{N-1}]^T$. In the next step, each data block is multiplied by U number of different phase sequences which can be denoted as

$P^{(u)} = [P_0^{(u)}, P_1^{(u)}, P_2^{(u)}, P_3^{(u)}, ...P_{N-1}^{(u)}]$, both the input data and phase sequence have the same length N. After multiplication, we get U no of modified data blocks. To include the unmodified data block in the set of modified data blocks, we set $P^{(1)}$ as the all-one vector of length N. Let us denote the modified data block for the uth phase sequence as

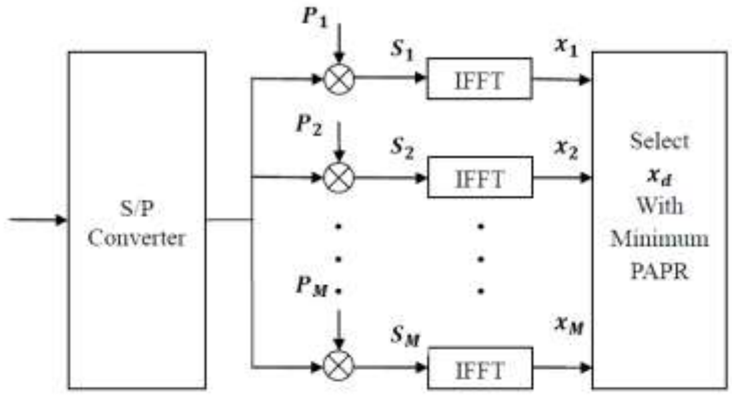


Fig. 1: Conventional SLM block diagram

$X^{(u)} = [X_0 P_{u,0}, X_1 P_{u,1}, X_2 P_{u,2}, X_3 P_{u,3}, \ldots.., X_{N-1} P_{u,N-1}]^T$ $u = 1, 2\ldots$ *U*. After applying SLM to **X**, the multicarrier signal becomes $x^{(u)}(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n P_{u,n}.e^{j2\pi n\Delta ft}, 0 < t < NT, u = 1, 2, \ldots U$ (10) Among the modified data blocks $X^{(u)}$, $u = 1, 2\ldots U$, the one with the lowest PAPR is selected for transmission. For implementation, the SLM technique needs *U* number of IDFT operations, and the number of required side information bits is $\log_2 U$ for each data block. This approach is applicable with all types of modulation and any number of subcarriers. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the design of the phase sequences.

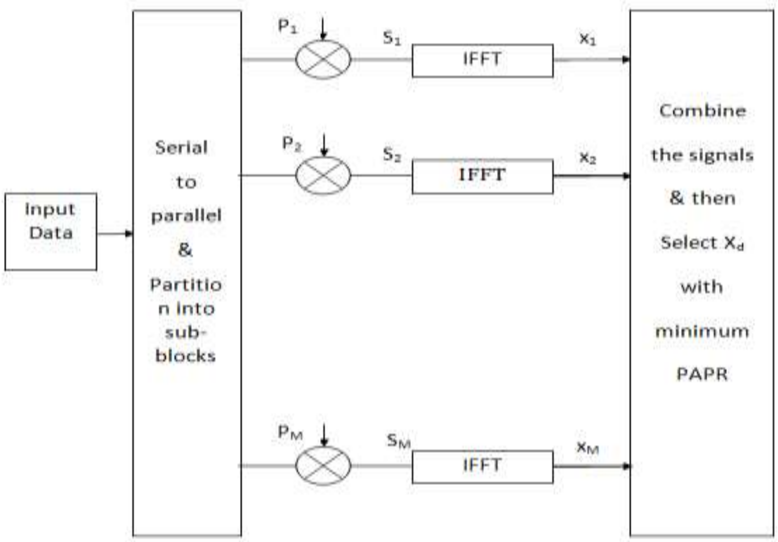


Fig. 2: Modified SLM scheme block diagram

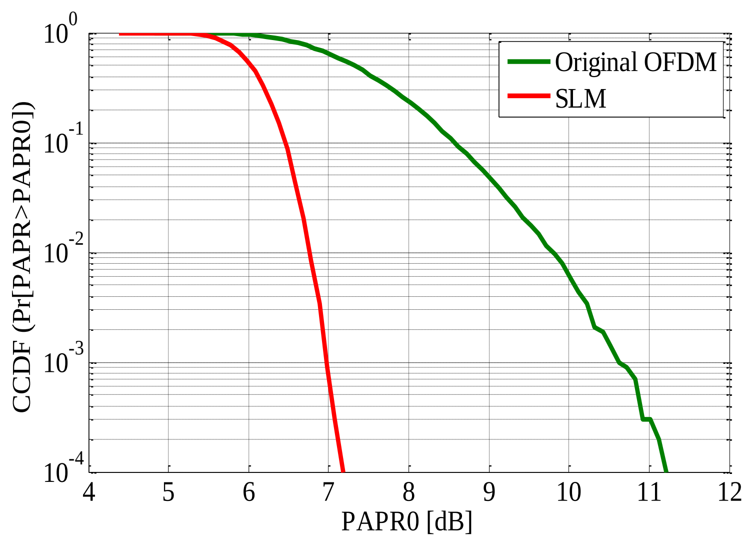
MODIFIED SLM SCHEME

Fig. 3 shows the block diagram of the proposed modified SLM scheme. In this modified scheme, first of all, a sufficient number of modulated data are generated and then data sets first are divided into a number of sub-blocks say $P_1, P_2, P_3, \ldots, P_M$. Then, the successive operations like in conventional scheme such as multiplication with the phase sequence, inverse fast Fourier transforms are done to the data.

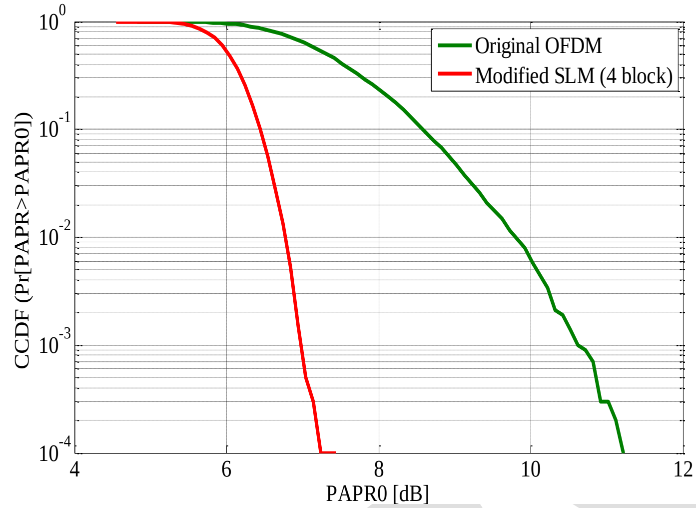
After phase sequence multiplication, we get $S_1, S_2, S_3, ..., S_M$. The Fourier transformed data are denoted as $x_1, x_2, x_3, ..., x_M$ at the end, these data are combined and their peak to average power ratio is calculated. The signal with lowest PAPR value is chosen. Dividing the generated data into sub-blocks and add them at end after several operations are some similar approaches that are applied in another signal scrambling technique called Partial Transmit Sequence (PTS) . But the main difference between these two techniques is that in case of PTS technique, after sub-dividation, instead of multiplying the partitions with phase sequences which is done in the proposed modified scheme, ifft operation is done and then the signal streams are multiplied with some assigned weighted values.

SIMULATIONS AND DISCUSSION

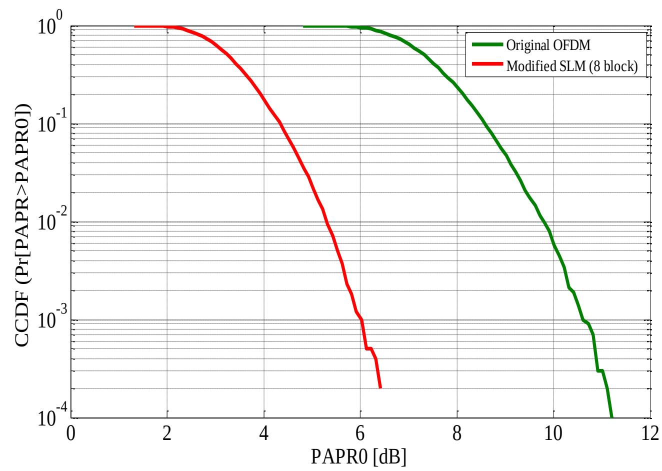
Matlab software has been used to plot the described conventional and modified schemes. Fig. 1 shows the PAPR performance of conventional SLM scheme. We take ten thousand data from the generated quadrature amplitude modulated data. The number of sub-bands we choose for our cases in this paper is 64. Phase factor possible values are taken as 1,-1, j, -j. The number of OFDM symbol candidates is sixteen with the mentioned sub-band numbers and the phase sequence. Data are oversampled with factor 4. Now, firstly, the PAPR of original PAPR is plotted using the equation (). The green solid line in Fig. 3 is indicating that original OFDM PAPR performance. The PAPR value (db) is found as 8.5, 9.8, 10.6, 11.2 for the CCDF values at .1,.01,.001,.0001 respectively. After applying SLM technique to the original signal, we get improved PAPR performance as 6.4,6.7,6.9,7.2 for the corresponding CCDF points. In case of our modified scheme, we applied sub-block partitioning. The simulation is conducted for 4, 8, and 16 sub-blocks. The 4-block scheme performance which is illustrated in Fig. 3(b), is remarkably less than the previous performance. For more number of sub-block partitioning like 8 and 16 sub-blocks in Fig. 3(c) and Fig. 3(d), even better performance with less PAPR value has been achieved. Values tabulated in Table I indicates the comparison of different simulated schemes



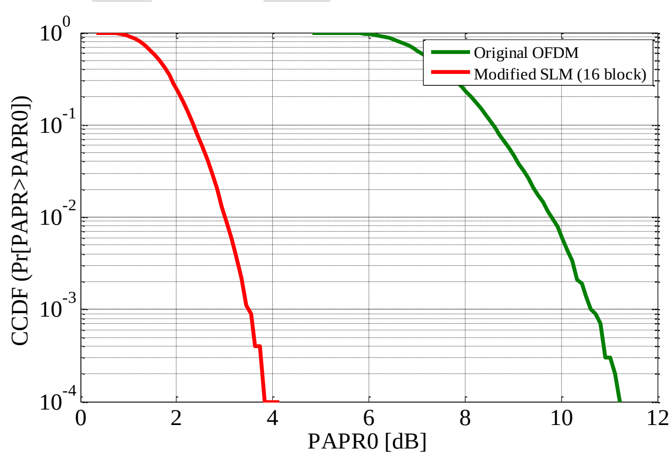
(a)



(b)



(c)



(d)

Fig. 3: PAPR reduction performance of different schemes (a) CCDFs of PAPR of conventional scheme (b) CCDFs of PAPR of modified scheme (4 block partitions) (c) CCDFs of PAPR of modified scheme (8 block partitions) (d) CCDFs of PAPR of modified scheme (16 block partitions)

TABLE I

PAPR VALUES AT DIFFERENT CCDF

	Normal scheme (equitional value)	Normal scheme (experimental value)	Modified scheme (4 sub blocks)	Modified scheme (8 sub blocks)	Modified scheme (16 sub blocks)
10^{-1}	8.58	6.455	4.89	4.33	2.4
10^{-2}	9.82	6.765	5.88	5.4	3.03
10^{-3}	10.62	6.98	6.7	5.9	3.4
10^{-4}	11.225	7.18	7.243	6.39	3.91

CONCLUSION

This paper presented a modified version of selected mapping technique to reduce PAPR of OFDM signals based on the idea of sub-block partitioning. The conducted simulations show that with the increment of the number of sub-block partitions, better result can be achieved though the computational complexity increases with the number of partitioning. Thus, it can be concluded that when the system is desired to be efficient in spectral transmission rather than cost effective ness, the proposed scheme can be a preferable one.

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