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An Inventory Model For Deteriorating Items With Two Parameter Weibull Deterioration And Price Dependent Demand

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Abstract - This paper presents an inventory model for deteriorating items with price dependent demand. Deterioration rate follows two parameter weibull distribution. Shortages are allowed and are completely backlogged.

Keywords: Deterioration items, holding cost, inventory, price-dependent demand time, shortages, weibull, completely backlogged.

INTRODUCTION:

Deterioration of items in an inventory is a common phenomenon in business situations. This is due to the fact that the items in the inventory become obsolete, devalued, decay or damaged depending on the type of goods. As a consequence of the deterioration shortages may occur. Hence deterioration factor has to be given importance while determining the optimal policy for an inventory model.

Ajanta Roy [1] presented an inventory model for time proportional deterioration rate and demand is function of selling price. The Author discussed the model without shortage and also with shortages in which the shortages are completely backlogged. Mukesh Kumar, Anand Chauhan, Rajat Kumar [9] extended Ajanta Roy models with trade credit. Tripathy C.K and L.M. Pradhan [14] gave a model in which the demand of the product decreases with the increase of time and sale price and deterioration rate follows a three parameter Weibull distribution functions. Tripathy C.K and L.M.Pradhan[15] gave a model in which the demand of the product decreases with the increase of time and sale price and deterioration follows a three parameter weibull distribution. Padmanabhan. G,Prem Vrat[10] formulated an EOQ model perishable items under stock dependent selling rate. Sahoo.N.K.,Sahoo.C.K. & Sahoo.S.K[12] described an inventory model for price dependent demand and time varying holding cost. Vikas Sharma and Rekha Rani Chaudhary [17] explained and inventory model for two parameter Weibull deterioration rate. They found profit for their model. Sanjay JAIN and Mukesh KUMAR [13] explained an inventory model with ramp type demand and three parameter Weibull deterioration rate. The Authors also analyzed and summarized economic order quantity models done by few researchers. There are some products which start deteriorate only after some interval of time. This was explained by taking three parameter Weibull distribution deterioration rate. Anil Kumar Sharma, Manoj Kumar Sharma and Nisha Ramani [3] described an inventory model for two – parameter Weibull distribution deterioration rate and demand rate is power pattern. Manoj Kumar Meher, Gobinda Chandra Panda, Sudhir Kumar Sahu [8] adopted a two – parameter Weibull distribution deterioration to develop an inventory model under permissible delay in payments. Kun – Shan Wu [7] made an attempt in his paper to obtain the optimal ordering quantity of deteriorating items for two – parameter Weibull distribution deterioration under shortages and permissible delay in payments. P.K.Tripathy and S.Pradham [16] also define an inventory model with two – parameter Weibull distribution as demand rate and deterioration rate increases with time. Recently R.Amutha and E.Chandrasekaran [2] developed an inventory model for deteriorating items with three-parameter weibull deterioration and price dependent demand.

In this present paper, we have developed an inventory model for two-parameter Weibull deterioration rate and price dependent demand. Shortages are allowed and are completely backlogged. Holding cost is assumed to be constant. Our aim is to increase the profit.

ASSUMPTIONS AND NOTATIONS

(i) The demand rate is a function of selling price.

(ii) Shortages are allowed and are completely backlogged

(iii) Lead time is zero.

(iv) Replenishment is instantaneous

(v) A is the set up cost

(vi) C is the unit cost of an item

(vii) p is the selling price

(viii) Demand D (t) = f (p) = a-p, where a > p.

(ix) C_2 is the shortage cost per unit time.

(x) $\theta(t) = \alpha\beta t^{\beta-1}$, $0 \le \alpha < 1$, $\beta > 0$ and $-\infty < \gamma < \infty$ is the deterioration rate. At time T_1 the Inventory becomes Zero and shortages start occurring.

(xi) h is the constant holding cost.

(xii) T is the length of the cycle.

MATHEMATICAL FORMULATION AND SOLUTION

Let I (t) be the inventory at time T ($0 \le t \le T$) the differential equation for the instantaneous state over (0,T) are given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a-p) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -(a-p) \qquad t_1 \le t \le T \qquad (2)$$

Solving equations (1) and (2) with boundary condition $I(t_1)$ we get

$$I(t) = -(a-p)\left[(t-t_1) + \frac{\alpha}{\beta+1}\left[t^{\beta+1} - t_1^{\beta+1}\right] + \frac{\alpha^2}{2(2\beta+1)}\left[t^{2\beta+1} - t_1^{2\beta+1}\right]\right] \qquad (3)$$

$$I(t) = -(a-p)(t-t_1)$$

Shortage cost (4)

$$SC = \frac{-C_2}{T}\int_{t_1}^{T} -(a-p)(t-t_1)dt$$

$$= \frac{C_2}{2T}(a-p)(t-t_1)^2 \qquad (5)$$

Holding cost

$$HC = \frac{h}{T}\int_0^{t_1} I(t)dt$$

$$= \frac{-h}{T}(a-p)\left(-\frac{t_1^2}{2} + \frac{\alpha}{\beta+1}\left[\frac{t_1^{\beta+2}}{\beta+2} - t_1^{\beta+2}\right] + \frac{\alpha^2}{2(2\beta+1)}\left[\frac{t_1^{2\beta+2}}{2\beta+2} - t_1^{2\beta+1}\right]\right) \qquad (6)$$

Stock loss due deterioration

$$D = (a-p)\int_0^{t_1} e^{\alpha t^\beta} dt - (a-p)\int_0^{t_1} dt$$

$$= (a-p)\left[\frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1^{2\beta+1}}{2(2\beta+1)}\right] \qquad (7)$$

Order quality

$Q = D + \int_0^T (a-p)dt$

$= (a-p)\left[\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\right]$ (8)

Purchase cost $= \frac{CQ}{T}$

$= \frac{(a-p)C}{T}\left[\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\right]$ (9)

Total profit per unit is = p(a-p) - $\frac{1}{T}$ [OC + SC + HC + PC]

$K(p,T,T_1) = p(a-p) - \frac{1}{T}\left[A + \frac{C_2}{2T}(a-p)(t-t_1)^2 - \frac{h}{T}(a-p)\left(\frac{-t_1^2}{2} + \frac{\propto}{\beta+1}\left[\frac{t_1^{\beta+2}}{\beta+2} - t_1^{\beta+2}\right] + \right.\right.$

$\left.\frac{\propto^2}{2(2\beta+1)}\left[\frac{t_1^{2\beta+2}}{2\beta+2} - t_1^{2\beta+1}\right]\right) + \frac{(a-p)C}{T}\left(\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\right)$ (10)

Let $t_1 = vT$, $0 < v < 1$

Therefore we have profit function,

$K(p,T) = p(a-p) - \frac{1}{T}\left[A + \frac{C_2}{2}(a-p)(t-vT)^2 - h(a-p)\left(-\frac{(vT)^2}{2} + \frac{\propto}{\beta+1}\left[\frac{(vT)^{\beta+2}}{\beta+2} - (vT)^{\beta+2}\right] + \frac{\propto^2}{2(2\beta+1)}\left[\frac{(vT)^{2\beta+2}}{2\beta+2} - (vT)^{2\beta+2}\right]\right) + \right.$

$\left. c(a-p)\left(\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT\right)\right]$ (11)

$\frac{\partial K(p,T)}{\partial T} = \frac{1}{T^2}\left[A + \frac{c_2}{2}(a-p)(t-vT)^2 + h(a-p)\left[\frac{(vT)^2}{2} + \frac{\propto}{\beta+2}(vT)^{\beta+2} + \frac{\propto^2}{2}\frac{(vT)^{2\beta+2}}{2\beta+2}\right]\right.$

$\left. + (a-p)c\left[\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT\right]\right] + -\frac{1}{T}\left[c_2(a-p)(vT-t)v + c(a-p)(\alpha v^{\beta+1}T^{\beta}\right.$

$\left. + \frac{\propto^2}{2} v^{2\beta+1} T^{2\beta} + v) + h(a-p)\left(v^2 T + \propto T^{\beta+1} v^{\beta+2} + \frac{\propto^2}{2} v^{2\beta+2} T^{2\beta+1}\right]\right.$ (12)

$\frac{\partial K(p,T)}{\partial p} = a - 2p + \frac{c_2}{2T}(t-vT)^2 - \frac{h}{T}\left[\frac{(vT)^2}{2} + \frac{\propto}{\beta+2}(vT)^{\beta+2} + \frac{\propto^2}{2(2\beta+2)}(vT)^{2\beta+2}\right] - \frac{c}{T}\left[\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT\right]$ (13)

For the maximization of profit we set

$\frac{\partial K(p,T)}{\partial T} = 0$ and $\frac{\partial K(p,T)}{\partial p} = 0$ provided $\frac{\partial^2 K(p,T)}{\partial T^2} < 0$, $\frac{\partial^2 K(p,T)}{\partial p^2} < 0$ and

$\left(\frac{\partial^2 K(p,T)}{\partial T^2}\right)\left(\frac{\partial^2 K(p,T)}{\partial p^2}\right)\left(\frac{\partial^2 K(p,T)}{\partial T \partial p}\right) > 0$.

CONCLUSION

A deterministic inventory model for deteriorating inventory model with two parameter Weibull distribution deterioration rate has been developed. Demand rate is function of selling price and holding cost is constant occurring shortages and completely backlogged.

REFERENCES:

[1] Ajanta Roy "An inventory model for deteriorating items with price dependent demand and time-varying holding cost." AMO-Advanced Modeling and Optimization, volume 10, November 1, 2008.

[2] Amutha.R, E.Chandrasekaran, "An inventory model for deteriorating items with three parameter weibull deterioration and price dependent demand" International Journal of Engineering Research & Technology Vol.2,Issue -5,May-2013,1931-1935.

[3] Anil Kumar Sharma, ManojKumar Sharma and Nisha Ramani "An Inventory model with Weibull distribution deteriorating power pattern demand with shortages and time dependent holding cost." American Journal of Applied Mathematics and Mathematical Sciences (Open Access Journal Volume 1, Number 1-2 January – December 2012, Pp. 17-22.

[4] Biplab Karmakar and Karabi Dutta Choudhury, "A Review on Inventory Models for Deteriorating Items with Shortages," Assam University Journal of Science& Technology, Physical Sciences and Technology, Vol.6, Number II, 51-59,2010

[5] Ghosh S.K and K.S.Chaudhri, "An order level inventory model for a deteriorating item with Weibull distribution deterioration time – quadratic demand and shortages", Advanced Modeling and Optimization, volume 6, Number 1.

[6] Goyal S.K, B.C.Giri "Recent trends in modeling of deteriorating inventory" European Journal of Operation Research, 134 (2001) 1-16

[7] Kun –Shan Wu "An Ordering Policy for Items with Weibull distribution Deterioration under permissible Delay in Payments." Tamsui Oxford Journal of Mathematical Science 14 (1998) 39-54.

[8] Manoj Kumar Meher , Gobinda Chandra Panda, Sudhir Kumar Sahu," An Inventory Model with Weibull Deterioration Rate under the Delay in Payment in Demand Dec ling Market" Applied Mathematical Sciences,Vol.6, 2012, no.23, 1121 -1133.

[9] Mukesh Kumar, Anand Chauhan, Rajat Kumar "A Deterministic Inventory Model for Deteriorating Items with Price Dependent Demand and Time Varying Holding cost under Trade Credit" International Journal of Soft computing and Engineering (IJSCE) ISSN:2231-2307, Volume-2, Issue-1, March 2012

[10] Padmanabhan.G., Prem Vrat "EOQ models for perishable items under stock dependent selling rate"

[11] Pattnaik.M, "A Note on Non Linear Optimal Inventory Policy Involving Instant Deterioration of Perishable Items with Price Discounts", The Journal of Mathematics and Computer SCienceVol.3, No.2 (2011) 145-155.

[12] Sahoo.N.K, Sahoo .C.K. & Sahoo.S.K. "An Inventory Model for Constant Deteriorating Items with Price Dependent Demand and Time p Varying Holding Cost" International Journal of computer cuience & communication Vol.1, No.1, January –June 2010, pp.267-271

[13] Sanjay JAIN, Mukesh KUMAR "An EOQ Inventory Model for Items with Ramp Type Demand, Three-Parameter Weibull Distribution Deterioration and Starting with Shortage", Yugoslav Journal Of Operations Research Volume 20(201), No. 2, 249-259

[14] Tripathy C.K and L.M. Pradhan, "Optimal Pricing and Ordering Policy for Three Parameter Weibull Deterioration .Int. Journal of math .Analysis, Vol.5,2011,no.6,275-284

[15] Tripathy C.K and L.M. Pradhan An EOQ model for three parameter Weibull deterioration with permissible delay in payments and associated salvage value," International Journal of Industrial Computations3(2012)115-122

[16] TripathyP.K, S.Pradhan, "An Integrated Partial Backlogging Inventory Model having Weibull Demand and Variable Deterioration rate with the Effect of Trade Credit", International Journal of Scientific & Engineering Research Volume 2, Issue 4, April 2011

[17] Vikas Sharma, and Rekha Rani Chaudhary, "An inventory Model for deteriorating items with Weibull Deterioration with Time Dependent Demand and Shortages", Research journal of Management sciences, Vol. 2(3), 28-30, March (2013) ISSN 2319-1171.