A COMPARATIVE STUDY OF DIFFERENT NOISE FILTERING TECHNIQUES IN DIGITAL IMAGES

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Abstract:

Although various solutions are available for de-noising them, a detail study of the research is required in order to design a filter which will fulfil the desire aspects along with handling most of the image filtering issues. In this paper we want to present some of the most commonly used noise filtering techniques namely: Median filter, Gaussian filter, Kuan filter, Morphological filter, Homomorphic Filter, Bilateral Filter and wavelet filter. Median filter is used for reducing the amount of intensity variation from one pixel to another pixel. Gaussian filter is a smoothing filter in the 2D convolution operation that is used to remove noise and blur from image. Kuan filtering technique transforms multiplicative noise model into additive noise model. Morphological filter is defined as increasing idempotent operators and their laws of composition are proved. Homomorphic Filter normalizes the brightness across an image and increases contrast. Bilateral Filter is a non-linear edge preserving and noise reducing smoothing filter for images. Wavelet transform can be applied to image de-noising and it has been extremely successful. Salt and pepper noise, Speckle noise and Gaussian noise are introduced to clean images and filtered using the filtering techniques mentioned above. The performance of the filtering techniques is evaluated based on signal to noise ratio. It is found that Wavelet based filter gives the best result amongst the chosen filtering techniques.

Keywords: Median Filter, Gaussian filter, Wiener Filter, Kuan filter, Wavelet transform, Bilateral filtering, Morphological Filtering, Homomorphic Filtering.

1. Introduction

1.1 Image processing:

Digital Image Processing is a component of digital signal processing .The area of digital image processing refers to dealing with digital images by means of a digital computer. Digital image processing is the process of enhancing samples of image which may or may not degraded by noise and other distortion.

There are some fundamental steps of image processing:

a. **Image Acquisition**: This is the first step or process of the fundamental steps of digital image processing. Image acquisition could be as simple as being given an image that is already in digital form. Generally, the image acquisition stage involves pre-processing, such as scaling etc.
b. **Image Enhancement**: Image enhancement is among the simplest and most appealing areas of digital image processing. Basically, the idea behind enhancement techniques is to bring out detail that is obscured, or simply to highlight certain features of interest in an image. Such as, changing brightness and contrast etc.
c. **Image Restoration**: Image restoration is an area that also deals with improving the appearance of an image. However, unlike enhancement, which is subjective, image restoration is objective, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image degradation.
d. **Morphological Processing**: Morphological processing deals with tools for extracting image components that are useful in the representation and description of shape.
e. **Segmentation**: Segmentation procedures partition an image into its constituent parts or objects.
f. **Object recognition**: Recognition is the process that assigns a label, such as, vehicle to an object based on its descriptors.
g. **Representation and Description**: Representation and description almost always follow the output of a segmentation stage, which usually is raw pixel data, constituting either the boundary of a region or all the points in the region itself. Choosing a representation is only part of the solution for transforming raw data into a form suitable for subsequent computer processing. Description deals with extracting attributes that result in some quantitative information of interest or are basic for differentiating one class of objects from another.
h. **Knowledge Base**: Knowledge may be as simple as detailing regions of an image where the information of interest is known to be located, thus limiting the search that has to be conducted in seeking that information.

1.2 Types of digital images:

a. **Binary**: In binary image the value of each pixel is either black or white. The image have only two possible values for each pixel either 0 or 1, we need one bit per pixel.
b. **Grayscale**: In grayscale image each pixel is shade of gray, which have value normally 0 [black] to 255 [white]. This means that each pixel in this image can be shown by eight bits that is exactly of one byte.
c. **True Color or RGB**: Each pixel in the RGB image has a particular color; that color in the image is described by the quantity of red, green and blue value in image. If each of the components has a range from 0255, this means that this gives a total of 2563 different possible colors values.

1.3 Noise in Images

Noise in image, is any degradation in an image signal, caused by external disturbance while an image is being sent from one place to another place via satellite, wireless and network cable.

Types of Image Noise:

a. **Salt and pepper noise**: It known as shot noise, impulse noise or Spike noise. Its appearance is randomly scattered white or black or both pixel over the image.
b. **Gaussian Noise**: Gaussian noise is caused by random fluctuations in the signal; it's modelled by random values added to an image. This noise has a probability density function [pdf] of the normal distribution. It is also known as Gaussian distribution.
c. **Speckle noise**: It can be modelled by random values multiplied by pixel values of an image.

2. Filtering Techniques

2.1 Median Filter:

2.1.1 Introduction:

Each imaging system suffers with a common problem of "Noise". Unwanted data which may reduce the contrast deteriorating the shape or size of objects in the image and blurring of edges or dilution of _ne details in the image may be term as noise [9].it may be due to one or more of the following reasons:

- ❖ Shortcomings of image acquisition devices
- ❖ Image developing mechanism
- ❖ Due to environment
- ❖ Physical nature of the system

Mathematically there are two basic models of Noise; additive and multiplicative. Additive noise is systematic in nature and can be easily modelled and hence removed or reduced easily [10]. Whereas multiplicative noise is image dependent, complex to model and hence difficult to reduce. When multiplicative noise caused by the de-phased echoes from the scattering appears, it is called Speckle Noise. Although it appears as noise but it contains useful information because it is due to surroundings of the target. Speckle may appear distinct in different imaging systems but it is always manifested in a granular pattern due to image formation under coherent waves.

2.1.2 Basic principle of median filter:

First we need to understand what a median filter is and what it does. In many different kinds of digital image processing, the basic operation is as follows:

At each pixel in a digital image we place a neighbourhood around that point, analyze the values of all the pixels in the neighbourhood according to some algorithm, and then replace the original pixel's value with one based on the analysis performed on the pixels in the neighbourhood [11]. The neighbourhood then moves successively over every pixel in the image, repeating the process.

The median filter is a sliding-window spatial filter. The median filter is normally used to reduce noise in an image, somewhat like the mean filter. However, it often does a better job than the mean filter of preserving useful detail in the image. This class of filter belongs to the class of edge preserving smoothing filters which are non-linear filters. This means that for two images A(x) and B(x)

$$\textbf{\textit{median}}[A(X) + B(X)]\ 6 = \textbf{\textit{median}}[A(X)] + \textbf{\textit{median}}[B(X)] \qquad \textbf{(2.1)}$$

These filters smooth the data while keeping the small and sharp details.

The median is just the middle value of all the values of the pixels in the neighbourhood. Note that this is not the same as the average (or mean); instead, the median has half the values in the neighbourhood larger and half smaller. The median is a stronger "central indicator" than the average. In particular, the median is hardly affected by a small number of discrepant values among the pixels in the neighbourhood. Consequently, median filtering is very effective at removing various kinds of noise. But it is special for "Salt and pepper noise" [12].

Like the mean filter, the median filter considers each pixel in the image in turn and looks at its nearby neighbours to decide whether or not it is representative of its surroundings. Instead of simply replacing the pixel value with the mean of neighbouring pixel values, it replaces it with the median of those values. The median is calculated by first sorting all the pixel values from the surrounding neighbourhood into numerical order and then replacing the pixel being considered with the middle pixel value. (If the neighbourhood under consideration contains an even number of pixels, the average of the two middle pixel values is used). A template of size 3x3, 5x5, 7x7, etc is applies to each pixel. The values within this template are sorted and the middle of the sorted list is used to replace the templates central pixel. Figure 2.1illustrates an example of median filtering.

2.1.3 Advantages of Median filter:

- No need to generate new pixel value.
- Easy to implement.
- Since the median is less sensitive than the mean to extreme values (outliers), those extreme values are more effectively removed.

2.1.4 Disadvantages of median filter:

- Median filter is not good for all types of noise, it is very good only for removing salt and Pepper noise.

2.2 Gaussian filter:

Gaussian filtering is used to blur images and remove noise and detail. In one dimension, the Gaussian function is:

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x^2}{2\sigma^2}} \quad (2.2)$$

Where σ is the standard deviation of the distribution The distribution is assumed to have a mean of 0.The Standard deviation of the Gaussian function plays an important role in its behaviour. The values located between +/- σ account for 68 % of the set, while two standard deviations from the mean account for 95 % and three standard deviations account for 99.7 %. This is very important when designing a Gaussian kernel of fixed length [8].

In probabilistic terms, it describes 100% of the possible values of any given space when varying from negative to positive values Gauss function is never equal to zero. It is a symmetric function.

2.2.1 Application of Gaussian filter:

- It defines a probability distribution for noise or data.
- It is used in mathematics.
- It is a smoothing operator.

2.2.2 Algorithm of Gaussian Filter:

- The filter can be understood as taking a pixel as the average value of its surrounding pixels. From value perspective, it's a smoothing. On graphic, it's a blur effect. The centre point will lose its detail. If the value range is very large, the blur effect is very strong.
- Normal distribution is an acceptable weight distribution model. On graphic, normal distribution is a Bell-shaped curve, the closer to the centre, the bigger the value.
- The normal distribution above is one dimensional, the graph is two dimensional. We need two dimensional normal distribution. The density function of normal distribution is called Gaussian function.
- To calculate the weight matrix, we need to set the value of σ
- With weight matrix, we can calculate the value of Gaussian Blur.
- If a point is at the border, there are not enough points, we need to copy all the existing points to respective places to form a new matrix [7].

2.2.3 Advantages of Gaussian filter

- Gaussian smoothing is very effective for removing Gaussian noise
- The weights give higher significance to pixels near the edge.
- Computationally efficient.
- Rotationally symmetric.

2.2.4 Disadvantages of Gaussian filter

- It takes much time.
- It reduces details.

2.3 Wiener Filter:

The Wiener filter purpose is to reduce the amount of noise present in a signal by comparison with an estimation of the desired noiseless signal. It is based on a statistical approach [2].

Typical filters are designed for a desired frequency response. The Wiener filter approaches filtering from a different angle. One is assumed to have knowledge of the spectral properties of the original signal and the noise, and one seeks the LTI filter whose output would come as close to the original signal as possible. Wiener filters are characterized by the following:

- Assumption: signal and (additive) noise are stationary linear stochastic processes with known spectral characteristics or known auto-correlation and cross-correlation.
- Requirement: the filter must be physically realizable, i.e. causal (this requirement can be dropped, resulting in a non-causal solution).
- Performance criteria: minimum mean-square error.

A useful approach to this filter-optimization problem is to minimize the mean-square value of the error signal that is defined as the difference between some desired response and the actual filter output. For stationary inputs, reduce the amount of noise present in a signal by comparison with an estimation of the desired noiseless signal [1].

The final details of the filter specification, however, depend on two other choices that have to be made:

- Whether the impulse response of the filter has finite or infinite duration.
- The type of statistical criterion used for the optimization.

1.3.1 Applications of Wiener Filter:

Wiener filters play a central role in a wide range of applications such as linear prediction, echo cancellation, signal restoration, channel equalization and system identification [1].

1.3.2 Steepest descent search algorithm for finding the Wiener FIR optimal filter:

Given the autocorrelation matrix

$$\mathbf{R = E[x(n)\,xt(n)]} \qquad (2.3)$$

The cross-correlation vector

$$\mathbf{p(n) = Eu(n)d(n)} \qquad (2.4)$$

Initialize the algorithm with an arbitrary parameter vector w(0). Iterate for

n = 0, 1, 2, 3. . . . . $n_m ax$

$$\mathbf{w(n+1) = w(n) + \mu[P — Rw(n)]} \qquad (2.5)$$

Stop iterations if

$$\mathbf{— P – Rw(n) — !\varepsilon}$$

1.3.3 Advantage of Wiener filter:

- Here we can use a large window to smooth the speckle noise.
- We can use a small window to avoid blurring edges such as field boundaries.

1.3.4 Disadvantage of Wiener Filter:

- It is difficult to estimate the power spectra.
- It is very difficult to obtain a perfect restoration for the random nature of the noise.
- Wiener filters are comparatively slow to apply since they require working in the frequency domain.

1.4 Kuan Filtering:

Speckle is the result of the diffuse scattering, which occurs when a sound wave (RF sound or Ultrasound) pulse randomly interferes with the small particles or objects on a scale comparable to the sound wavelength. Speckle is an inherent property of SAR and Ultrasound images, and is modelled as spatial correlated multiplicative noise. In most cases, it is considered a contaminating factor that severely degrades image quality. Medical imaging like Ultrasound is very popular due to its low cost, least harmful to human body, real time view and small size. But this imaging has major disadvantage of having Speckle [13].Synthetic Aperture Radar (SAR) is an active sensor that uses microwave signals for transmission and it detects the wave that is reflected back by the objects. SAR is widely used for obtaining high resolution images of the earth. It is used in the fields of remote sensing, oceanography, geology, ecology etc. Pixels in the image represent the back scattered radiation from an area in the imaged scene. Brighter areas are produced by stronger radar responses and darker areas are from weaker radar responses. Consider an original image Y corrupted by the Multiplicative noise. The resultant distorted image X, may be written as:

$$\mathbf{X(i, j) = h(i, j)\ \_\ Y(i, j);} \qquad (2.6)$$

Where Y (i, j) is Original image, h(i, j) is noise and X(i, j) is resultant noisy image.

1.4.1 Basic principle of Kuan filter:

Kuan Filtering technique transforms multiplicative noise model into additive noise model. Let us denote L the filter window cantered at pixel coordinates (x,y), CL the centre pixel in L, and W a weighting function. The gray level of the Kuan filtered pixel f at position (x,y) is given by:

$$\mathbf{f(x; y) = \mu_L + W(CL — \mu_L)} \qquad (2.7)$$

Where μ_L is the local mean within the filter window L. The weighting function in a Kuan filter is defined as [14].

$$\mathbf{W} = \frac{1-\frac{c_{u}2}{C_{i}2}}{1+c_{u}2}; \qquad (2.8)$$

Where $C_i = \frac{\sigma_L}{\mu_L}$, σ_L the standard deviation in L, and Cu = $\frac{1}{\sqrt{NLOOK}}$

The NLOOK parameter effectively controls the amount of smoothing applied to the image by varying the noise variation coefficient Cu. Theoretically, the value of NLOOK should be the Effective Number of Looks (ENL) of the image. It should be close to the actual number of looks of the noisy image, but may be different if the image has undergone re sampling. A smaller NLOOK value leads to more smoothing and a larger NLOOK value preserves more image features (e.g. edges). The user may experimentally adjust the NLOOK value so as to control the effect of the filter [15]. The ENL can be estimated from a uniform image region A by:

$$\mathbf{ENL} = \left(\frac{\mu_A}{\sigma_A}\right)^2 \qquad (2.9)$$

Where μ_A and σ_A are the mean and standard deviation of A, respectively.

1.4.2 Advantages of Kuan filter:

- It does not make an approximation on the noise variance within the filter window.
- Kuan filter simply models the multiplicative model of speckle into an additive linear form.

1.4.3 Disadvantage of Kuan filter:

- It relies on the Effective Number of Looks (ENL) from a SAR image to determine a different weighting function W.

1.5 Wavelet transform for de-noising:

The application of the wavelet transform to image de-noising has been extremely successful. Nowadays, it has been used in image processing, data compression, and signal processing. Here we will discuss how thresholding is applied to de-noise an image in wavelet domain. In conventional Fourier transform, we use sinusoids for basic functions. It can only provide the frequency information. Temporal information is lost in this transformation process. In some applications, we need to know the frequency and temporal information at the same time, such as a musical score, we want to know not only the notes (frequencies) we want to play but also when to play them. Unlike conventional Fourier transform, wavelet transforms are based on small waves, called wavelets. It can be shown that we can both have frequency and temporal information by this kind of transform using wavelets. Moreover, images are basically matrices. For this reason, image processing can be regarded as matrix processing. Due to the fact that human vision is much more sensitive to small variations in color or brightness, that is, human vision is more sensitive to low frequency signals. Therefore, high frequency components in images can be compressed without distortion. Wavelet transform is one of a best tool for us to determine where the low frequency area and high frequency area is .When we look at some images, generally we see many regions (objects) that are formed by similar texture. If the objects are small in size, we normally examine them at high resolutions. Contrary we examine big objects at low resolutions. This is the fundamental motivation for multi-resolution processing [19].

We can create a multi resolution pyramid of images at each level, we just store the differences (residuals) between the image at that level and the predicted image from the next level we can reconstruct the image by just adding up all the residuals.

1.5.1 Wavelet transform:

Wavelets are families of functions $\boldsymbol{\psi_{s,t}(x)}$ generated from a single base wavelet $\boldsymbol{\psi(x)}$ by dilations and translations

$$\boldsymbol{\psi_{s,t}(x) = \frac{1}{\sqrt{s}}\psi\left(\frac{x-t}{s}\right) \qquad s \neq 0} \qquad \textbf{(....1)} \quad \textbf{(2.10)}$$

Where s is the dilation (scale) parameter and t is the translation parameter. Wavelets must have mean zero, and the useful ones have localized support in both spatial and Fourier domains. There are orthogonal and non-orthogonal wavelet sets that span $L^2(R)$.It is very close to the derivative of a Gaussian function. The set of $\psi_{m,n}(x)$ spans $L^2(R)$ when $s = 2^n$; t = n.

$$\boldsymbol{\psi_{m,n}(x) = 2^{\frac{-m}{2}}\psi(2^{-m}(x-n))} \qquad \textbf{(....2)} \quad \textbf{(2.11)}$$

Where m is the scale index (m = 0,1,2,. . .), and n is the translation (spatial) index (n = . . . , -2, -1,0, 1,2,. . .). The discrete wavelet transform W(m, n) of a 1-D function f(x) is defined as the projection of the function onto the wavelet set $\psi_{m,n}(x)$.

$$\boldsymbol{W(m, n) = \int_{-\propto}^{\propto} f(x)\psi_{m,n}(x)dx} \qquad \textbf{(....3)} \quad \textbf{(2.12)}$$

Since the set of $\psi_{m,n}(x)$ spans the space containing *f(x)*, the reconstruction of function *f(x)* from its wavelet transform *W(m, n)* is possible. The wavelet coefficients measure how closely correlated the wavelet is with each section of the signal [1].

$$\boldsymbol{F(x) = \sum_m \sum_n \psi'_{m,n}(x)W(m,n)} \qquad \textbf{(....4)} \quad \textbf{(2.13)}$$

Where $\boldsymbol{\psi'_{m,n}(x)}$ is the normalized dual basis of $\boldsymbol{\psi_{m,n}(x)}$ for the wavelet expansion we use here $\boldsymbol{\psi' \approx \psi}$. The wavelet transform ***W(m, n)*** gives a scale-space decomposition of signals and, with simple modifications, images. It decomposes the signal into different resolution scales, with m indexing the scale and n indexing position in the original signal space. In practice, we are concerned with a finite length, discrete (sampled), 1-D data set *f(k); k = 1, 2, ...N* and we need appropriate discrete and finite versions of the calculations involved in the wavelet decomposition [9]. In particular, there is a fixed limit to the resolution and, therefore, a lower bound on the scale index m, which we may take as m = 1 without loss of generality. It is useful to model this resolution limit by representing the data *f(k)* as samples of a smoothed, or low-passed, version of a continuous signal.

$$f(k) = \int_{-\propto}^{\propto} f(x)\emptyset'(x-k) \qquad (....5) \quad (2.14)$$

With respect to a smoothing or scaling function a. Based on this representation of the data, one may compute the wavelet coefficients in (3) by means of a purely discrete algorithm, as detailed in [9]. Beyond these considerations, there is also an effective upper limit on the scale m imposed by the finite length of the signal. Consequently, the non-orthogonal, discrete, dyadic wavelet coefficients *W(m, n)* are computed on a 2-D space of m = 1, 2, ..., M-1 and n = 1, 2, ..., N where M = $\log_2 N$ with the remaining information contained in the coarse scale averages

$$S(M,n) = 2^{-M}\int_{-\propto}^{\propto} f(x)\emptyset\left(2^{-M}(x-n)\right)dx \qquad (2.15)$$

This information determines the signal *f(x):*

$$f(x) = \sum_{m=1}^{M-1}\sum_{n=1}^{N}\Psi'_{m,n}(x)W(m,n) + \sum_{n=1}^{N}\emptyset\left(2^{-M}(x-n)\right)S(M,n) \qquad (....6) \quad (2.16)$$

The M.N coefficients obviously form an over complete representation of the signal. For a data set of N = 256 points, M is equal to 8, i.e., there are eight wavelet scales. At each scale, there are 256 data points corresponding to the signal detail projected at that scale [16].

1.5.2 Wavelet based image compression by decomposition:

- The wavelet transforms decomposes an image into a set of different resolution sub images corresponding to the various frequency bands.
- The analysis filter consists of high pass filter and low pass filter.
- The low pass filter (average operation) extracts the coarse information.
- While High pass filter (differencing operation) extracts detail information [21].

1.5.3 The noise reduction based on wavelet analysis:

One of the important applications of wavelet analysis is for noise reduction. Wavelet transform provides a powerful tool for the solution to this problem, with its good time-frequency localized character [18]. At present, the basic methods of wavelet noise reduction are:

- Using wavelet transform modulus maxima: according to signal and noise on different scales of modulus maxima of different propagation characteristics, select signal modulus maxima in all wavelet transform modulus maxima, remove the modulus maxima of the noise, then reconstruct signal with the remaining wavelet transform modulus maxima. This method can retain the singular point information of signal effectively while reducing the noise.
- Based on the correlation of wavelet coefficients of each scale: there is correlation among the wavelet coefficients of each scale, and the correlations between the signal and the noise of each scale is different. The correlation of the signal wavelet transform of each scale is obvious, while that of the noise wavelet transform is not obvious. Retain the wavelet transform coefficient in the large scale and high correlation coefficient in the small scale, leave the weak correlation coefficient, and then reconstruct the signal. This method can acquire the steady noise reduction effect.
- Using the nonlinear wavelet transform threshold method: it is generally thought that after the wavelet transform, the relatively large amplitude wavelet coefficients is mainly the signal, while the relatively small amplitude wavelet coefficients is mainly the noise. Noise reduction with threshold is to decompose noisy signal, retain the wavelet coefficients greater than the proper threshold set in advance and adjust the wavelet coefficients less than the proper threshold to zero. Finally, reconstruct the signal with the treated wavelet coefficients. This method can restrain the white noise of the signal completely [22].

1.5.4 Wavelet Threshold Decreasing noise method:

The model of a one-dimensional noisy signal can be noted down as Eq. (3).

$$s(t) = f(t) + \sigma e(t) \qquad (....7) \quad (2.17)$$

Where *f(t)* is the useful signal, *e(t)*noise, σ-noise level and *s(t)*the signal under consideration. In most cases it suggested that the function *e(t)* is described by the white (Gaussian) noise model, which meets the condition *N(0,1)* and the noise level for one. In actual project, information about the noise is contained in the high frequency spectral region of the signal, while the useful information is contained in the low frequency one or some of the stable one. Therefore, the signal noise reduction process can be handled as follows: first make the signal wavelet decomposition, such as carrying out three-tier decomposition and the process of decomposition. Noise is usually part of the cd1, cd2, cd3. We can treat the wavelet coefficients with the threshold, and then reconstruct the signal to meet the purpose of the noise reduction. Because the threshold method can get the approximate optimal estimation of original signal, calculate speedily and adapt widely, it is one of the most widely used wavelet decreasing-noise method. Threshold decreasing-noise method is applied mainly to the signal mixed with white noise; using threshold decreasing-noise method has the advantage of almost completely restraining the noise and well as reserving the peak point reecting the characteristics of the original signal. Soft threshold decreasing-noise method can even make the decreasing- noise signal the approximate optimal estimation of original signal and the estimated signal and the original signal are at least the same smooth without additional oscillation [17].

Generally, the wavelet threshold decreasing noise method of signal can be divided into the following three steps:

- Wavelet decomposition of signal. Choose a wavelet and determine the level N of wavelet decomposition, then according to the corresponding wavelet, from the noisy signal, obtain the high frequency coefficients decomposed in each order.
- Threshold quantification of the high frequency coefficients of wavelet decomposition. Quantify the threshold of the high frequency coefficients in the corresponding order after 1- N order scale decomposition, to get a new high frequency coefficient of wavelet.
- Wavelet reconstruction. According to the N-order low-frequency coefficients got from wavelet decomposition and the high-frequency 1 – N order coefficients after threshold processing, synthesis and reconstruct the signal with wavelet to get the noised signal.

In these three steps, the key is how to choose the threshold and how to quantify the threshold, to a certain degree, it relates to the quality of the noise reduction of signal.

At present, there are a variety of threshold selection methods: VisuShrink, RiskShrink, Sure- Shrink, WaveJSShrink and so on. One of the most common method is VisuShrink, which introduces the overall unified threshold $\lambda=\sigma\sqrt{2logN}$, where σ is the standard deviation of the noise signal, N is the signal length.

Choose a threshold to quantify the each high frequency coefficient of wavelet decomposition. Make *cd(j)* as the original high-frequency coefficient of decomposition scale, and as the threshold quantified high-frequency coefficient. During the threshold quantification of wavelet decomposition in the high frequency coefficients, threshold function embodies the different treatment strategies and quantitative methods to the high-frequency coefficients module exceeded or below the threshold [20].

Define:

$$\boldsymbol{cd(j)} = \begin{cases} \boldsymbol{cd(j)}, \ |\boldsymbol{cd(j)}| \geq \boldsymbol{\lambda(j)} \\ \boldsymbol{0}, \ |\boldsymbol{cd}\ \boldsymbol{(j)}| \leq \ | \leq \boldsymbol{\lambda(j)} \end{cases} \qquad \textbf{(....8)} \quad \textbf{(2.18)}$$

as hard threshold quantification method;

Define:

$$\boldsymbol{cd(j)} = \begin{cases} \boldsymbol{sign(cd(j))}, \ (\ |\boldsymbol{cd(j)}| - \boldsymbol{\lambda(j)}), \ |\boldsymbol{cd(j)}| \geq \boldsymbol{\lambda(j)} \\ \qquad\qquad \boldsymbol{0}, \ |\boldsymbol{cd}\ \boldsymbol{(j)}| \leq \ | \leq \boldsymbol{\lambda(j)} \end{cases} \qquad \textbf{(....9)} \quad \textbf{(2.19)}$$

as soft threshold quantification method.

1.5.5 Main algorithm for Wavelet Threshold Noise Reduction:

There are generally three approaches to threshold with the wavelet analysis:

- The default threshold decreasing-noise. This method uses function dencmp to generate the default threshold of signal, then use function wdencmp for the noise reduction.
- Given threshold decreasing-noise. In actual decreasing-noise process, the threshold can be usually obtained through experience formula, and the credibility of such threshold is higher than that of the default threshold. Function wthresh is available when conducting threshold quantification processing.
- Mandatory decreasing-noise. Adjust all of the high-frequency coefficients of the wavelet decomposition to zero, which means removing all of the high-frequency part, and then reconstruct the signal with wavelet. This method is relatively simple and decreasing-noise signal is smooth, but it easily loses the useful components of the signal.

1.5.6 Comparison of hard and soft thresholding:

- It is known that soft thresholding provides smoother results in comparison with the hard thresholding.
- More visually pleasant images, because it is continuous.
- Hard threshold, however, provides better edge preservation in comparison with the soft one.
- Sometimes it might be good to apply the soft threshold to few detail levels, and the hard to the rest.

1.6 Bilateral filtering:

The idea underlying bilateral filtering is to do in the range of an image what traditional filters do in its domain. Two pixels can be close to one another, that is, occupy nearby spatial location, or they can be similar to one another, that is, have nearby values, possibly in a perceptually meaningful fashion. Closeness refers to vicinity in the domain, similarity to vicinity in the range. Traditional filtering is domain filtering, and enforces closeness by weighing pixel values with coefficients that fall of with distance. Similarly, we define range filtering, which averages image values with weights that decay with dissimilarity. Range filters are nonlinear because their weights depend on image intensity or colour. Computationally, they are no more complex than standard non-separable filters. Most importantly, they preserve edges.

Spatial locality is still an essential notion [24]. In fact, we show that range filtering by itself merely distorts an image's colour map. We then combine range and domain filtering, and show that the combination is much more interesting. We denote the combined filtering as bilateral filtering. Since bilateral filters assume an explicit notion of distance in the domain and in the range of the image function, they can be applied to any function for which these two distances can be defined. In particular, bilateral filters can be applied to colour images just as easily as they are applied to black-and-white ones.

1.6.1 Image Smoothing with Gaussian Convolution:

Blurring is perhaps the simplest way to smooth an image; each output image pixel value is a weighted sum of its neighbours in the input image. The core component is the convolution by a kernel which is the basic operation in linear shift-invariant image filtering. At each output pixel position it estimates the local average of intensities, and corresponds to low-pass filtering. An image filtered by Gaussian Convolution is given by:

$$\boldsymbol{GC[I]_p = \sum_{q\in s} G_\sigma(\|p-q\|) I_q} \qquad \textbf{(1)} \qquad \textbf{(2.20)}$$

Where G_σ denotes the 2D Gaussian kernel.

$$\boldsymbol{G_\sigma(x) = \frac{1}{2\pi\sigma^2} exp(\frac{-x^2}{2\sigma^2})} \qquad \textbf{(2)} \qquad \textbf{(2.21)}$$

Gaussian filtering is a weighted average of the intensity of the adjacent positions with a weight decreasing with the spatial distance to the centre position p. The weight for pixel q is defined by the Gaussian $G_\sigma(\| p - q\|)$, where σ is a parameter defining the neighbourhood size. The strength of this influence depends only on the spatial distance between the pixels and not their values. For instance, a bright pixel has a strong influence over an adjacent dark pixel although these two pixel values are quite different. As a result, image edges are blurred because pixels across discontinuities are averaged together.

1.6.2 Edge-preserving Filtering with the Bilateral Filter:

The bilateral filter is also defined as a weighted average of nearby pixels, in a manner very similar to Gaussian convolution. The difference is that the bilateral filter takes into account the difference in value with the neighbours to preserve edges while smoothing. The key idea of the bilateral filter is that for a pixel to influence another pixel, it should not only occupy a nearby location but also have a similar value. The bilateral filter, denoted by *BF [ ]*, is

$$\boldsymbol{BF[I]_p = \frac{1}{W_p}\sum_{q\in s} G_{\sigma_s}(\| p-q \|) G_{\sigma_s}(\mid I_p - I_q \mid) I_q} \qquad \textbf{(3)} \qquad \textbf{(2.22)}$$

Where normalization factor W_p ensures pixel weights sum to 1.0:

$$\boldsymbol{W_p = \sum_{q\in s} G_{\sigma_s}(\| p-q \|) G_{\sigma_s}(\mid I_p - I_q \mid)} \qquad \textbf{(4)} \qquad \textbf{(2.23)}$$

Parameters σ_s and σ_r will specify the amount of filtering for the image I. Equation (3) is a normalized weighted average where $\boldsymbol{G_{\sigma_s}}$ is a spatial Gaussian weighting that decreases the influence of distant pixels, σ_r is a range Gaussian that decreases the influence of pixels q when their intensity values differ from I_p.

1.6.3 Parameters:

The bilateral filter is controlled by two parameters: σ_s and σ_r .

- As the range parameter σ_r increases, the bilateral filter gradually approximates Gaussian convolution more closely because the range Gaussian $\boldsymbol{G_{\sigma_s}}$ widens and flattens, i.e., is nearly constant over the intensity interval of the image.
- Increasing the spatial parameter σ_s smooth's larger features.

An important characteristic of bilateral filtering is that the weights are multiplied: if either of the weights is close to zero, no smoothing.

Iterations: The bilateral filter can be iterated. This leads to results that are almost piece-wise constant. Although this yields smoother images, the effect is different from increasing the spatial and range parameters. Increasing the spatial parameters σ_s has a limited effect unless the range parameter σ_r is also increased. Although a large σ_r also produces smooth outputs, it tends to blur the edges whereas iterating preserves the strong edges.

Separation: The bilateral filter can split an image into two parts: the filtered image and its 'residual' image. The filtered image holds only the large-scale features, as the bilateral filter smoothed away local variations without affecting strong edges. The residual image, made by subtracting the filtered image from the original, holds only the image portions that the filter removed. Depending on the settings and the application, this removed small-scale component can be interpreted as noise or texture [22].

1.7 Morphological Filtering:

1.7.1 Mathematical Morphology:

Mathematical morphology mostly deals with the mathematical theory of describing shapes using sets. In image processing, mathematical morphology is used to investigate the interaction between an image and a certain chosen structuring element using the basic operations of erosion and dilation. Mathematical morphology stands somewhat apart from traditional linear image processing, since the basic operations of morphology are non-linear in nature, and thus make use of a totally different type of algebra than the linear algebra [27].

1.7.2 Morphology and Sets:

Morphology is formulated in terms of set theory. Sets represent objects in image. Morphological processing is constructed with operations on sets of pixels. Basically it is constructed for binary images. The set of all white pixels in a binary image is a complete morphological description of an image [5]. In binary images, the sets are members of the 2D integer space Z2, where each element of a set is a tuple (2D vector) whose coordinates are the (x,y) coordinates of a white (or black) pixel in the image. The same can be extended for grey-scale images. Gray-scale images can be represented as sets, whose components are in Z3: two components are coordinates of a pixel and the third its discrete intensity value.

Morphological operations for binary images provide a basic techniques and those operations for gray scale images requires more sophisticated mathematical concepts for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, etc [28].

1.7.3 Structuring Elements:

The structuring elements are small set of sub images used to probe an analyzed image for properties of interest. The structuring element has both a shape and an origin.

Depending on shape, structuring element can take additional parameters. There are two types of structuring element, i) Flat Structuring Element, ii) Non-at Structuring Element. Basically, Flat structuring element are used for two dimensional image and Non at structuring element for three-dimensional object, the parameter added for Non-at i.e. height. The structuring element like: Disk, Diamond, Arbitrary shape, Pair, Periodic line, Rectangle, Line, Disk and Octagon are belongs to at structuring element, whereas Arbitrary and Ball are the non-at Structuring element [29].

1.7.4 Morphological processing operations:

More formal descriptions and examples of how basic morphological operations works are given below:

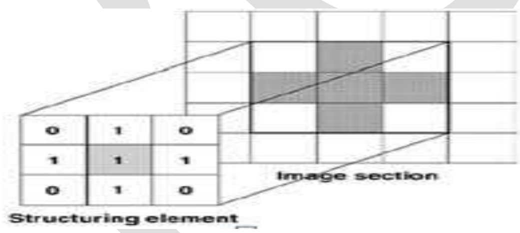

Figure: Example of a structuring element

1.7.5 Erosion and dilation:

Dilation grows or enlarges objects in a binary image. The manner and extend of this growth image is controlled by the structuring element.

$X \oplus B = \{P \in Z^2 \mid P = x + b, x \in X, b \in B\}$ **(2.24)**

This equation is based on reflecting B about its origin and shifting this reflection by z. The dilation of X by B then is the set of all displacements z, such that and X overlap by at least one element.

However, the preceding equations are more intuitive when viewing the structural element as a convolution mask. Dilation is based on set operations and therefore is a nonlinear operation, while the convolution is linear.

The disadvantage of dilation operation is that during the time of removing the noisy pixels inside the foreground or object region, the boundary of the object region also gets increased.

Erosion shrinks or removes objects in a binary image. With X and B as sets, the erosion of X by B is defined as

$X \ominus B = \{P \in Z^2 \mid P + b \in X \,\forall\, b \in B\}$ **(2.25)**

Where P is the Translation member.

The erosion of X by B is the set of all points z such that B, translated by z, is contained in X, where set B is a structuring element.

In image processing applications, dilation and erosion are used most often in various combinations. An image undergoes a series of dilations and/or erosions using the same or different structuring elements.

1.7.6 Opening and Closing:

Opening generally smoothes the contour of an object and eliminate thin protrusions. The opening of a set X by structuring element B is defined as:

$$\boldsymbol{X \bullet B = (X \ominus B) \oplus B} \qquad \textbf{(2.26)}$$

Therefore, the opening X by B is the erosion of X by B, followed by a dilation of the result by B.

Closing also tends to smooth sections of contours but fusing narrow breaks and long, thin gulfs and eliminating small holes and filling gaps in the contour. The closing of a set X by structuring element B is defined as:

$$\boldsymbol{X \bullet B = (X \oplus B) \ominus B} \qquad \textbf{(2.27)}$$

Therefore, the closing X by B is the dilation of X by B, followed by an erosion of the result by B.

1.7.7 Hit or Miss Transform:

The **hit or miss transformation** is useful to match specified configurations of pixels in an image, such as isolated foreground pixels, or pixels that are endpoints of line segments.

One of the important applications of this operation is used in forensics. Employment of fingerprints as evidence of crime has been one of the most important utilities in forensics since 19th century. Where there are no witness to a certain crime, finger prints can be very useful in determining the offenders. In most cases, they are incomplete and degraded. The individual features that uniquely identify a fingerprint are called minutiae. Thus, the basic ridge pattern together with the minutiae and their location on the finger print pattern uniquely identify a fingerprint. The Morphological Image Processing enhances the degraded noisy and / or incomplete latent fingerprints.

In physics and related fields, computer techniques routinely enhance images of experiments in areas such as high-energy plasmas and electron microscopy. Similarly successful applications of image processing concepts can be found in astronomy, biology, nuclear medicine, law enforcement, and defence.

1.7.8 Applications of Gray scale morphology:

Gray morphological operations are used generally to extract the edge of the image for the conditions, the multi-level geodetic expansion to expand to fill the target area, through which the algorithm can improve the detection of image accuracy, enhanced noise immunity, effectively identify the target and also to efficiently reduce the number of skeleton points and the entropy of morphological decomposition.

- Blood vessel edge enhancement and reconnection.
- Geographic pattern recognition for a satellite remote sensing image.
- Recognition and classification of vehicle on the traffic road in the hi-resolution satellite image.
- Solution to problem of luminal contour detection in intravascular ultrasound images.
- A robust vision system for vehicle license plate recognition Industrial parts recognition and inspection by image morphology.
- Multiresolutional texture analysis based on morphological techniques.
- Image matching using morphological operations.
- Extraction of grid patterns on stamped metal sheets.
- Application of Morphological Operations in Human Brain CT Image with SVM.
- Morphological detection based on size and contrast criteria application to cells detection.

1.8 Homomorphic Filtering:

1.8.1 Introduction:

If the image model is based on illumination-reflectance, then frequency domain procedures are not as easy to perform. The main reason is that illumination and reflectance components of the model are not separable. To be able to improve appearance of an image by simultaneous brightness range compression and contrast enhancement it is necessary to separate the two components [32]. As you recall, an image can be modelled mathematically in terms of illumination and reflectance as follow:

$$\boldsymbol{F(x.y) = I(x,y) * R(x,y)} \qquad \textbf{(2.28)}$$

Where $*$ is the multiplicative noise High Pass is one such technique for removing the multiplicative noise. Before applying high pass filtering we generally take the logarithmic values of the both the sides. i.e.

$$\boldsymbol{ln(F(x;\, y)) \;=\; ln(I(x;\, y) \,*\, R(x;\, y))} \qquad \textbf{(2.29)}$$

Now we very well know from the logarithmic properties that if we take log of both the sides then we can express log(intensity) as the sum of log(illumination) and log(reflectance) i.e.

$$\boldsymbol{ln(F(x;\, y)) \;=\; ln(I(x;\, y)) \;+\; ln(R(x;\, y))} \qquad \textbf{(2.30)}$$

The Fourier transformed signal is processed by means of a filter function H(u,v) and the resulting function is inverse Fourier transformed. Finally, inverse exponential operation yields an enhanced image. This enhancement approach is termed as homomorphic filtering [33].

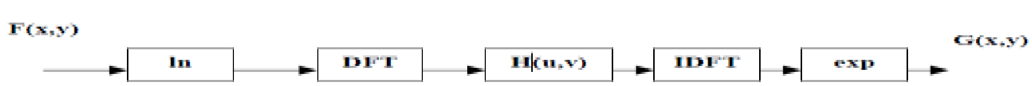

Figure: Operation of Homomorphic filtering

1.8.2 Operation steps of Homomorphic filtering:

Following are the steps for Homomorphic filtering:

- We will first convert the image to float.
- Convert the image next to the log domain
- Apply the High pass filtering in either the spatial domain or the frequency domain
- Apply inverse filter and retain the real part of the result
- Apply the exponential function to invert the log transform
- See the homomorphic filtered image using ifftshow() function

2. Conclusion

In practical applications, the original noise-free image is not available and only the noisy version exists. We have presented a comparative study between Median Filter, Kuan filter, Wiener filter, Gaussian filter, Wavelet Threshold filter, Bilateral filter, Morphological filter, Homomorphic filter, Gaussian noise, Speckle noise. All these filtering techniques performance can be evaluated on the basis of SNR. Through which we will be able to get our best image filtering techniques.

In future work we are planning to evaluate the performance of all the filtering techniques using Signal Noise Ratio and planning to implement soft computing based noise filter for digital images and comparatively evaluate the performance with the filtering techniques mentioned in this paper.

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