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A Fuzzy based Two Warehouse Inventory model for deteriorating items with Cubic Demand and different fuzzy cost parameters

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Abstract— This paper deals with a fuzzy based two warehouse inventory model for decaying items in which demand is taken to be cubic function of time, holding cost, ordering cost, backorder costs which are fuzzy in nature. Shortages are allowed in the owned warehouse. Due to lack of historical data and information the probability of certain new products are not known and in that case to remove certain uncertainty fuzzy concept is used. In this context we have used triangular fuzzy numbers to represent the fuzzy parameters. Finally the model is solved mathematically and profit maximization technique is used to illustrate the system using Signed Distance method.

Keywords— Inventory, Own warehouse, Rented warehouse, Cubic demand, Fuzzy model, Signed distance method, Triangular fuzzy number, Holding cost.

INTRODUCTION

An inventory system deals with the decision that minimizes the net cost of the system and maximizes the net profit of the system. In the present competitive business world many of the inventory models are formulated with the unrealistic assumption that all produced items are of good quality. But it is impossible for the production company to produce all the good quality products as there always remains some defective items .In an inventory model the cost parameters such as holding cost, ordering cost, purchase cost, demand etc are known and have definite value without ambiguity. Some of the business fit such conditions , but in most of the cases due to the changing market scenario the parameters are imprecise. This uncertainty concept can be defined as fuzziness or vagueness. The authority have to decide how much and when to order or manufacture so that the profit should be maximum and total cost associated with the inventory system should be minimum. Also in reality there are so many physical goods which deteriorate during the stock in periods due to different factors like dryness, damage, spoilage and vaporization. Deterioration means worsening of products. It is a natural phenomenon in our daily life. Products like vegetables, milk, domestic goods, fashion goods; electronic components etc are deteriorating items. So this deteriorating must be considered to formulate an inventory model.

During the last few decades Inventory models have been widely applied in the competitive business world. Ghare and Schrader(1963) developed for the first time an inventory model for deteriorating items. Convert and Philip(1973) extended their work. Hartely (1976) first proposed a problem in his book “Operations Research – A Managerial Emphasis”. In the formulation, the transportation cost for transferring the items from *RW* to *OW* was not considered. After Hartely (1976), Sarma (1983) extended the model under the assumption that stocks of *RW* are transferred from *RW* to *OW* in a bulk release fashion with fixed transportation cost per unit. Dave (1988) discussed the two-warehouse inventory models for finite and infinite rate of replenishment rectifying the errors of the model developed by Sarma (1983) and gave analytical solution of each model. Further, Goswami and Chaudhuri (1992) developed the models with and without shortages for linearly time dependent demand. In their formulation, stocks of *RW* are transferred to *OW* in equal time interval. Correcting and modifying the assumptions of Goswami and Chaudhuri (1992), Bhunia and Maiti (1994) discussed the same model and graphically presented the sensitivity analyses on the optimal average cost and the cycle length for the variations on the location and shape parameters of demand. The same type of model was developed by, Kar et al. (2001), Zhou and Yang (2003) and Mondal et al. (2007) for different types of demand. Donaldson (1977) developed an optimal algorithm for solving classical no shortage inventory model. Sarma (1987) developed a two-warehouse inventory model with infinite replenishment and completely backlogged shortages. Dave (1989) proposed a deterministic lot size inventory model with shortages and a linear trend in demand. Goswami and Chaudhuri (1991) discussed different types of inventory models with linear trend in demand. Pakkala and Achary (1992) then extended Sarma’s (1987) model for finite replenishment rate. All the models in Sarma (1987) and Pakkala and Achary (1992) were developed for prescribed scheduling period (cycle length), uniform demand and stocks of *RW* transferred to *OW* in continuous release fashion ignoring the transportation cost for this purpose. Benkherouf (1997) presented a two-warehouse model for deteriorating items with the general form of time dependent demand under continuous release fashion. Bhunia and Maiti (1997) discussed the same type of problem considering linearly (increasing) time dependent demand with completely backlogged shortages. This model was developed for infinite time horizon with the repetition of entire cycle with the changed value of the location parameter of the time dependent demand. Recently, very few researchers developed this type of model considering finite time horizon. All these models mentioned earlier were discussed only for non-deteriorating items. Lee and Ma (2000) developed a no-shortage inventory model for perishable items with free form of time dependent demand and fixed planning horizon. In their model, some cycles are of single warehouse system and the remaining is of two-warehouse system. Kar et al. (2001) discussed two storage inventory problems for non-perishable items with linear trend in demand over a fixed planning horizon considering lot-size dependent replenishment cost (ordering cost). On the other hand, considering two-storage facilities, Yang (2004) developed two inventory models for deteriorating items with uniform demand rate and completely backlogged shortages under inflation. Recently, Yang (2006) extended the models introduced in Yang (2004) by incorporating the partially backlogged shortages. In the year 2007, Chung and Huang (2007) proposed two-warehouse inventory model for deteriorating items. Deb Choudhury.P and Dutta.P developed a two warehouse inventory model considering demand as cubic function of time. However, all these models were developed based on an impractical assumption that the rented warehouse has unlimited capacity.

The concept of fuzzy logic was first proposed by Zadeh(1965) .Bellam and Zadeh(1970) discussed the difference between randomness and fuzziness . Silver and Peterson(1985) developed decision systems for inventory management and production planning. Zimmermann(1985) gave a review on applications of fuzzy set theory. Park (1987) discussed the EOQ model in which trapezoidal fuzzy numbers are used. Yao and Lee(1999) presented a fuzzy inventory model with and without backorder for fuzzy order quantity with trapezoidal fuzzy number.

A deterministic inventory model for deteriorating items with two warehouses is developed. Inventory cost in rented warehouse is higher than that of own warehouse. Demand is taken cubic (time dependent) in nature. Holding costs, ordering cost and backorder cost are considered as fuzzy numbers. The triangular fuzzy numbers are used to represent the fuzzy parameters. The total profit of the system is obtained with the help of Sign distance defuzzification method.

PRELIMINARIES:

FUZZY SET

A fuzzy set $\breve{A}$ in a universe of discourse X is defined as the set of pairs $\breve{A} = \{(x, \mu_{\breve{A}}(x)) : x \in X\}$, where $\mu_{\breve{A}}(x)$:X→ [0,1] is a mapping and $\mu_{\breve{A}}(x)$ is called membership function of $\breve{A}$ or grade of membership of x in $\breve{A}$.

CONVEX FUZZY SET

A fuzzy set $\breve{A}$ in a universe of discourse is called convex if for all

$x_1, x_2 \in X, \mu_{\breve{A}}(\delta x_1 + (1-\delta)x_2 \geq \min\{\mu_{\breve{A}}(x_1), \mu_{\breve{A}}(x_2)\}, where\ \delta \in [0,1]$.

NORMAL FUZZY SET

A fuzzy set $\breve{A}$ is called normal fuzzy set if there exists at least one $x \in X$ such that $\mu_{\breve{A}}(x_1) = 1$.

FUZZY NUMBER

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature .But it actually represents the notation of a set of real numbers ‘closer to a’ where ‘a’ is the number being fuzzified. A fuzzy number is a fuzzy set which is both convex and normal.

TRIANGULAR FUZZY NUMBER (TFN)

A triangular fuzzy number $\breve{A}$ is represented by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function where $\mu_{\breve{A}}(x)$:X→ [0,1] is given by

$$\mu_{\breve{A}}(x) = f(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases},$$

ASSUMPTIONS & NOTATIONS

The following assumptions and notations have been used in developing the model:

1. Replenishment rate is infinite and lead time is constant.
2. Holding cost, demand both depends on time.
3. Shortages are allowed and unsatisfied demand in backlogged at a rate **A+Bx**, where A is constant, B is backlogging parameter (positive) and x is the waiting time.
4. Inventory cost in rented warehouse is higher than that of own warehouse.
5. Demand is assumed as, $D = a + bt + ct^2 + dt^3, where\ a, b, c, d > 0$
6. Holding cost in own warehouse assumed is h_o

   Holding cost in rented warehouse assumed is h_r . These parameters are fuzzy in nature.

7. C_s =fuzzy backlogging cost per unit per unit time
8. R =opportunity cost per unit
9. T =The length of the replenishment cycle
10. t_1 =time at which the inventory level falls to zero in RW
11. t_2 =time at which the inventory level falls to zero in OW
12. RW : Rented warehouse
13. OW : Own warehouse
14. α : deterioration rate in OW
15. β : deterioration rate in RW, $0 \leq \alpha, \beta < 1$
16. $I_1(t)$: Inventory level in RW at time t
17. $I_2(t)$: Inventory level in OW at time t
18. $I_3(t)$: The level of negative inventory at time t.
19. C : The purchase cost per unit.
20. P : Fuzzy Ordering cost.
21. q : capacity of own warehouse
22. p : selling price per unit, where p>C
23. Q : The ordering quantity
24. S : The maximum inventory level per cycle.

THE MATHEMATICAL MODEL

The model begins at time t=0.Initially the business community purchases a certain amount of item from market. From which certain amount is used to meet up the backorder quantities in previous cycle and 'q' units of products are kept in OW and the remaining amount in RW. During the time interval $0 \leq t \leq t_1$ the inventory level in RW decreases due to both demand and deterioration and becomes zero at t=t_1.But in OW the inventory level q decreases during $0 \leq t \leq t_1$,due to deterioration only and during $t_1 \leq t \leq t_2$ due to both demand and deterioration. At time $t = t_2$, the inventory level in OW reaches to zero. Then shortages are allowed to occur during $t_2 \leq t \leq T$ .Our objective is to find the maximum total average profit by considering all the relevant costs per unit time of the inventory system.

The inventory levels in RW and OW are given by the following differential equations

$$\frac{dI_1}{dt}+\beta I_1= -(a+bt+ct^2+dt^3) \quad , 0\leq t\leq t_1. \qquad \text{......... (1)}$$

With the condition I_1 (t) = 0, at t=t_1

And $$\frac{dI_2}{dt}+\alpha I_2=0 \qquad , 0\leq t\leq t_1. \qquad \text{.......... (2)}$$

With the condition I_2 (0) =q

The solution of (1) and (2) are,

$$I_1= - [a+b (t_1-\frac{1}{\beta}) +c (t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta^2})+d (t_1^3- \frac{9t_1^2}{\beta} +\frac{36t_1}{\beta^2} - \frac{60}{\beta^3})]\, e^{\beta (t_1-t)} + [a+b (t- \frac{1}{\beta}) +c (t^2 - \frac{4t}{\beta} + \frac{6}{\beta^2})+d (t^3- \frac{9t^2}{\beta} +\frac{36t}{\beta^2} - \frac{60}{\beta^3})] \qquad \text{............ (3)}$$

$$I_2= qe^{-\alpha t} \qquad \text{.................. (4)}$$

Again during the time $t_1\leq t\leq t_2$, the inventory level in OW decreases due to both demand and deterioration.

The differential equation involved here is:-

$$\frac{dI_2}{dt}+\alpha I_2= - (a+bt+ct^2+dt^3) \quad , t_1\leq t\leq t_2$$

With the condition I_2 (t) = 0, at t=t_2

Therefore,

$$I_2=-[a+b (t_2-\frac{1}{\alpha}) +c (t_2^2 - \frac{4t_2}{\alpha} + \frac{6}{\alpha^2})+d (t_2^3- \frac{9t_2^2}{\alpha} +\frac{36t_2}{\alpha^2} - \frac{60}{\alpha^3})]\, e^{\alpha (t_2-t)} + [a+b (t- \frac{1}{\alpha}) +c (t^2 - \frac{4t}{\alpha} + \frac{6}{\alpha^2})+d (t^3- \frac{9t^2}{\alpha} +\frac{36t}{\alpha^2} - \frac{60}{\alpha^3})] \qquad \text{.............. (5)}$$

From (4) and (5) and due to the continuity at t= t_1 is,

$$I_2 (t_1) = qe^{-\alpha t_1}$$

$$\Longrightarrow -[a+b (t_2-\frac{1}{\alpha}) +c (t_2^2 - \frac{4t_2}{\alpha} + \frac{6}{\alpha^2})+d (t_2^3- \frac{9t_2^2}{\alpha} +\frac{36t_2}{\alpha^2} -\frac{60}{\alpha^3})]\, e^{\alpha (t_2-t_1)} +[a+b (t_1- \frac{1}{\alpha}) +c (t_1^2 - \frac{4t_1}{\alpha} + \frac{6}{\alpha^2})+d (t_1^3- \frac{9t_1^2}{\alpha} +\frac{36t_1}{\alpha^2} - \frac{60}{\alpha^3})] = qe^{-\alpha t_1}$$

$\Longrightarrow$ q= $(e^{\alpha t_1} - e^{\alpha t_2})$ $[a+b\,(t_1- t_2) +c\,(t_1^2- t_2^2 - \frac{4t_1}{\alpha} + \frac{4t_2}{\alpha}) +d\,(t_1^3- t_2^3+ \frac{9}{\alpha}(t_2^2- t_1^2) + \frac{36}{\alpha^2}(t_1- t_2)]$

$$\Longrightarrow e^{\alpha t_1} - e^{\alpha t_2} = \frac{q}{[a+b\,(t_1- t_2) +c\,(t_1^2- t_2^2 - \frac{4t_1}{\alpha} + \frac{4t_2}{\alpha}) +d\,(t_1^3- t_2^3+ \frac{9}{\alpha}(t_2^2- t_1^2) + \frac{36}{\alpha^2}(t_1- t_2)]}$$

$$\Longrightarrow e^{\alpha t_2} = e^{\alpha t_1} - \frac{q}{[a+b\,(t_1- t_2) +c\,(t_1^2- t_2^2 - \frac{4t_1}{\alpha} + \frac{4t_2}{\alpha}) +d\,(t_1^3- t_2^3+ \frac{9}{\alpha}(t_2^2- t_1^2) + \frac{36}{\alpha^2}(t_1- t_2)]}$$

$$\Longrightarrow t_2 = \frac{\ln\left(e^{\alpha t_1} - \frac{q}{[a+b\,(t_1- t_2) +c\,(t_1^2- t_2^2 - \frac{4t_1}{\alpha} + \frac{4t_2}{\alpha}) +d\,(t_1^3- t_2^3+ \frac{9}{\alpha}(t_2^2- t_1^2) + \frac{36}{\alpha^2}(t_1- t_2)]}\right)}{\alpha} \qquad \text{.................................. (6)}$$

Again during, $t_2 \le t \le T$, shortage occurs, so inventory level is backlogged follows the differential equation

$\frac{dI_3}{dt} = -(a+bt+ct^2+dt^3)\,(A+Bt),\ t_1 \le t \le T$.................. (7)

With the condition $I_3\,(t) = 0$, at $t=t_2$

Therefore,

$I_3=Aa\,(t_2-t)+(\frac{aB}{2} + \frac{Ab}{2})\,(t_2^2- t^2) +(\frac{Ac}{3} + \frac{Bb}{3})\,(t_2^3- t^3) + (\frac{Bc}{4}+\frac{dA}{4})\,(t_2^4- t^4) + \frac{Bd}{5}(t_2^5- t^5)$ ....(8)

The order quantity about replenishment cycle is given as,

$Q = I_1\,(0) + I_2\,(0) - I_3\,(T)$

$=-[a+b\,(t_1- \frac{1}{\beta}) +c\,(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta^2}) +d\,(t_1^3- \frac{9t_1^2}{\beta} +\frac{36t_1}{\beta^2} -\frac{60}{\beta^3})]\,e^{\beta t_1} +[\,a - \frac{b}{\beta} + \frac{6c}{\beta^2} - \frac{60d}{\beta^3}] +q -[Aa\,(t_2-T)+(\frac{aB}{2} + \frac{Ab}{2})\,(t_2^2- T^2) +(\frac{Ac}{3} + \frac{Bb}{3})\,(t_2^3- T^3) + (\frac{Bc}{4}+\frac{dA}{4})\,(t_2^4- T^4) + \frac{Bd}{5}(t_2^5- T^5)]$............................ (9)

The maximum inventory level per cycle is,

$S = I_1\,(0) + I_2\,(0)$

$= -[a+b\,(t_1- \frac{1}{\beta}) +c\,(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta^2}) +d\,(t_1^3- \frac{9t_1^2}{\beta} +\frac{36t_1}{\beta^2} -\frac{60}{\beta^3})]\,e^{\beta t_1} +[\,a - \frac{b}{\beta} + \frac{6c}{\beta^2} - \frac{60d}{\beta^3}] +q$ .... (10)

Holding cost per cycle in RW,

$C_{H1} = \int_0^{t_1} h_r I_1 dt$

$= h_r\{[a + b\left(t_1 - \frac{1}{\beta}\right) + c\left(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta 2}\right) + d\left(t_1^3 - \frac{9t_1^2}{\beta} + \frac{36t_1}{\beta 2} - \frac{60}{\beta 3}\right)][\frac{1}{\beta} - \frac{e^{\beta t_1}}{\beta}] + (a - \frac{b}{\beta} - \frac{60d}{\beta^3} + \frac{6c}{\beta^2})t_1 + (b- \frac{4c}{\beta} + \frac{36d}{\beta^2})\frac{t_1^2}{2} + (c- \frac{9d}{\beta})\frac{t_1^3}{3} + d\frac{t_1^4}{4}\}$.................(11)

Holding cost per cycle in OW,

$C_{H2} = \int_0^{t_1} h_o I_2\,dt + \int_{t_1}^{t_2} h_o I_2\,dt$

$=-\frac{h_o q e^{-\alpha t_1}}{\alpha}+h_o\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta(t_2-t_1)}}{\beta}]+(a-\frac{b}{\alpha}-\frac{60d}{\alpha^3}+\frac{6c}{\alpha^2})(t_2-t_1)+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})(\frac{t_2^2}{2}-\frac{t_1^2}{2})+(c-\frac{9d}{\beta})(\frac{t_2^3}{3}-\frac{t_1^3}{3})+d(\frac{t_2^4}{4}-\frac{t_1^4}{4})\}$............(12)

Total holding cost, $C_H = C_{H_1}+C_{H_2}$ .... (13)

Ordering cost =P

Backlogging cost per cycle, SC= $-C_s\int_{t_2}^{T}(A+Bt)dt$

$=-C_s[(T-t_2)\,A+B\,(\frac{T^2-{t_2}^2}{2})]$................. (14)

Opportunity cost, OC = R $(T-t_2)[1-A-\frac{B}{2}(T+t_2)]$ .... (15)

Purchase cost per cycle, PC =CQ................ (16)

Sales revenue per cycle,

SR = p $[\int_0^{t_2}(a+bt+ct^2+dt^3)dt+\int_{t_2}^{T}(a+bt+ct^2+dt^3)\,(A+Bt)dt]$

$=p(at_2+b\frac{t_2^2}{2}+c\frac{t_2^3}{3}+d\frac{t_2^4}{4})+p[Aa(T-t_2)+(aB+bA)(\frac{T^2}{2}-\frac{{t_2}^2}{2})+(bB+Ac)(\frac{T^3}{3}-\frac{{t_2}^3}{3})+(Bc+Ad)(\frac{T^4}{4}-\frac{{t_2}^4}{4})+Bd(\frac{T^5}{5}-\frac{{t_2}^5}{5})]$ ....(17)

Total profit per unit is,

X = $\frac{1}{T}$ [SR-Ordering cost- C_H-OC-SC-PC]

$=\frac{1}{T}\{p(at_2+b\frac{t_2^2}{2}+c\frac{t_2^3}{3}+d\frac{t_2^4}{4})+p[Aa(T-t_2)+(aB+bA)(\frac{T^2}{2}-\frac{{t_2}^2}{2})+(bB+Ac)(\frac{T^3}{3}-\frac{{t_2}^3}{3})+(Bc+Ad)(\frac{T^4}{4}-\frac{{t_2}^4}{4})+Bd(\frac{T^5}{5}-\frac{{t_2}^5}{5})]-P-h_r\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta t_1}}{\beta}]+(a-\frac{b}{\beta}-\frac{60d}{\beta^3}+\frac{6c}{\beta^2})t_1+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})\frac{t_1^2}{2}+(c-\frac{9d}{\beta})\frac{t_1^3}{3}+d\frac{t_1^4}{4}\}-\frac{h_o q e^{-\alpha t_1}}{\alpha}+h_o\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta(t_2-t_1)}}{\beta}]+(a-\frac{b}{\alpha}-\frac{60d}{\alpha^3}+\frac{6c}{\alpha^2})(t_2-t_1)+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})(\frac{t_2^2}{2}-\frac{t_1^2}{2})+(c-\frac{9d}{\beta})(\frac{t_2^3}{3}-\frac{t_1^3}{3})+d(\frac{t_2^4}{4}-\frac{t_1^4}{4})\}+C_s[(T-t_2)A+B(\frac{T^2-{t_2}^2}{2})]-R(T-t_2)[1-A-\frac{B}{2}(T+t_2)]-C[-\{a+b\,(t_1-\frac{1}{\beta})+c\,(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta^2})+d(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta^2}-\frac{60}{\beta^3})\}\,e^{\beta t_1}+[a-\frac{b}{\beta}+\frac{6c}{\beta^2}-\frac{60d}{\beta^3}]]-Cq-C[Aa(t_2-T)+\frac{aB}{2}(t_2^2-T^2)+\frac{Ab}{2}(t_2^2-T^2)+\frac{Bb}{3}(t_2^3-T^3)+\frac{Ac}{3}(t_2^3-T^3)+\frac{Bc}{4}(t_2^4-T^4)+\frac{dA}{4}(t_2^4-T^4)+\frac{Bd}{5}(t_2^5-T^5)]$..(18)

Now, the total profit of the system is maximum if it satisfies

$\frac{\partial X}{\partial t_2}=0$, $\frac{\partial X}{\partial T}=0$; $\frac{\partial^2 X}{\partial {t_2}^2}>0$, $\frac{\partial^2 X}{\partial T^2}>0$

$\frac{\partial^2 X}{\partial {t_2}^2}\frac{\partial^2 X}{\partial T^2}-(\frac{\partial^2 X}{\partial t_2 \partial T})^2<0$

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FUZZY MODEL

In the above model we have developed a crisp model in which holding costs in RW and OW are fuzzy in nature along with ordering cost and backorder cost. We have considered these cost parameters as triangular fuzzy numbers and then defuzzified by the method of Signed distance defuzzification method.

SIGNED DISTANCE METHOD

Here holding costs, ordering cost, backorder cost are considered as triangular fuzzy number. Suppose the following numbers

(i) $h_r \in [h_r - \Delta_1, h_r + \Delta_2], 0 < \Delta_1 < h_r, 0 < \Delta_1\Delta_2$

(ii) $h_o \in [h_o - \Delta_3, h_o + \Delta_4], 0 < \Delta_3 < h_o, 0 < \Delta_3\Delta_4$

(iii) $P \in [P - \Delta_5, P + \Delta_6], 0 < \Delta_5 < P, 0 < \Delta_5\Delta_6$

(iv) $C_s \in [C_s - \Delta_7, C_s + \Delta_8], 0 < \Delta_7 < C_s, 0 < \Delta_7\Delta_8$

The signed distance method of the above fuzzy numbers are as

(i) $d(\widetilde{h_r}, 0) = h_r + \frac{1}{4}(\Delta_2 - \Delta_1)$

(ii) $d(\widetilde{h_o}, 0) = h_o + \frac{1}{4}(\Delta_4 - \Delta_3)$

(iii) $d(\check{P}, 0) = P + \frac{1}{4}(\Delta_6 - \Delta_5)$

(iv) $d(\widetilde{C_s}, 0) = C_s + \frac{1}{4}(\Delta_8 - \Delta_7)$

Now, $\check{X} = (X_1, X_2, X_3)$

$$X_1 = \frac{1}{T} \{ p(at_2 + b\frac{t_2^2}{2} + c\frac{t_2^3}{3} + d\frac{t_2^4}{4}) + p[Aa(T-t_2) + (aB+bA)(\frac{T^2}{2} - \frac{t_2^2}{2}) + (bB+Ac)(\frac{T^3}{3} - \frac{t_2^3}{3}) + (Bc+Ad)(\frac{T^4}{4} - \frac{t_2^4}{4}) + Bd(\frac{T^5}{5} - \frac{t_2^5}{5})] - (P-\Delta_5) - (h_r - \Delta_1)\{[a + b(t_1 - \frac{1}{\beta}) + c(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta2}) + d(t_1^3 - \frac{9t_1^2}{\beta} + \frac{36t_1}{\beta2} - \frac{60}{\beta3})][\frac{1}{\beta} - \frac{e^{\beta t_1}}{\beta}] + (a - \frac{b}{\beta} - \frac{60d}{\beta^3} + \frac{6c}{\beta^2})t_1 + (b - \frac{4c}{\beta} + \frac{36d}{\beta^2})\frac{t_1^2}{2} + (c - \frac{9d}{\beta})\frac{t_1^3}{3} + d\frac{t_1^4}{4}\} - \frac{(h_o-\Delta_3)qe^{-\alpha t_1}}{\alpha} + (h_o - \Delta_3)\{[a + b(t_1 - \frac{1}{\beta}) + c(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta2}) + d(t_1^3 - \frac{9t_1^2}{\beta} + \frac{36t_1}{\beta2} - \frac{60}{\beta3})][\frac{1}{\beta} - \frac{e^{\beta(t_2-t_1)}}{\beta}] + (a - \frac{b}{\alpha} - \frac{60d}{\alpha^3} + \frac{6c}{\alpha^2})(t_2 - t_1) + (b - \frac{4c}{\beta} + \frac{36d}{\beta^2})(\frac{t_2^2}{2} - \frac{t_1^2}{2}) + (c - \frac{9d}{\beta})(\frac{t_2^3}{3} - \frac{t_1^3}{3}) + d(\frac{t_2^4}{4} - \frac{t_1^4}{4})\} + (C_s - \Delta_7)[(T - t_2)A + B(\frac{T^2 - t_2^2}{2})] - R(T - t_2)[1 - A - \frac{B}{2}(T + t_2)] - C[-\{a + b(t_1 - \frac{1}{\beta}) + c(t_1^2 - \frac{4t_1}{\beta} + \frac{6}{\beta^2}) + d(t_1^3 - \frac{9t_1^2}{\beta} + \frac{36t_1}{\beta^2} - \frac{60}{\beta^3})\} e^{\beta t_1} + [a - \frac{b}{\beta} + \frac{6c}{\beta^2} - \frac{60d}{\beta^3}]] - Cq - C[Aa(t_2-T) + \frac{aB}{2}(t_2^2 - T^2) + \frac{Ab}{2}(t_2^2 - T^2) + \frac{Bb}{3}(t_2^3 - T^3) + \frac{Ac}{3}(t_2^3 - T^3) + \frac{Bc}{4}(t_2^4 - T^4) + \frac{dA}{4}(t_2^4 - T^4) + \frac{Bd}{5}(t_2^5 - T^5)]\}$$

$X_2 = X$

$$X_3=\frac{1}{T}\{ p(at_2+b\frac{t_2^2}{2}+c\frac{t_2^3}{3}+d\frac{t_2^4}{4})+p[Aa(T-t_2)+(aB+bA)(\frac{T^2}{2}-\frac{t_2^2}{2})+(bB+Ac)(\frac{T^3}{3}-\frac{t_2^3}{3})+(Bc+Ad)(\frac{T^4}{4}-\frac{t_2^4}{4})+Bd(\frac{T^5}{5}-\frac{t_2^5}{5})]-(P+\Delta_6)-(h_r+\Delta_2)\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta t_1}}{\beta}]+(a-\frac{b}{\beta}-\frac{60d}{\beta^3}+\frac{6c}{\beta^2})t_1+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})\frac{t_1^2}{2}+(c-\frac{9d}{\beta})\frac{t_1^3}{3}+d\frac{t_1^4}{4}\}-\frac{(h_o+\Delta_4)qe^{-\alpha t_1}}{\alpha}+(h_o+\Delta_4)\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta(t_2-t_1)}}{\beta}]+(a-\frac{b}{\alpha}-\frac{60d}{\alpha^3}+\frac{6c}{\alpha^2})(t_2-t_1)+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})(\frac{t_2^2}{2}-\frac{t_1^2}{2})+(c-\frac{9d}{\beta})(\frac{t_2^3}{3}-\frac{t_1^3}{3})+d(\frac{t_2^4}{4}-\frac{t_1^4}{4})\}+(C_s+\Delta_8)[(T-t_2)A+B(\frac{T^2-t_2^2}{2})]-R(T-t_2)[1-A-\frac{B}{2}(T+t_2)]-C[-\{a+b(t_1-\frac{1}{\beta})+c(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta^2})+d(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta^2}-\frac{60}{\beta^3})\}e^{\beta t_1}+[a-\frac{b}{\beta}+\frac{6c}{\beta^2}-\frac{60d}{\beta^3}]]-Cq-C[Aa(t_2-T)+\frac{aB}{2}(t_2^2-T^2)+\frac{Ab}{2}(t_2^2-T^2)+\frac{Bb}{3}(t_2^3-T^3)+\frac{Ac}{3}(t_2^3-T^3)+\frac{Bc}{4}(t_2^4-T^4)+\frac{dA}{4}(t_2^4-T^4)+\frac{Bd}{5}(t_2^5-T^5)]\}\text{.........(19)}$$

Total profit per unit time by signed distance method is as follows,

$$d(\check{X})=X+\frac{1}{T}\{ p(at_2+b\frac{t_2^2}{2}+c\frac{t_2^3}{3}+d\frac{t_2^4}{4})+p[Aa(T-t_2)+(aB+bA)(\frac{T^2}{2}-\frac{t_2^2}{2})+(bB+Ac)(\frac{T^3}{3}-\frac{t_2^3}{3})+(Bc+Ad)(\frac{T^4}{4}-\frac{t_2^4}{4})+Bd(\frac{T^5}{5}-\frac{t_2^5}{5})]-(\frac{\Delta_6-\Delta_7}{4})-(\frac{\Delta_2-\Delta_1}{4})\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta t_1}}{\beta}]+(a-\frac{b}{\beta}-\frac{60d}{\beta^3}+\frac{6c}{\beta^2})t_1+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})\frac{t_1^2}{2}+(c-\frac{9d}{\beta})\frac{t_1^3}{3}+d\frac{t_1^4}{4}\}-\frac{(\Delta_4-\Delta_3)qe^{-\alpha t_1}}{4\alpha}+(\frac{\Delta_4-\Delta_3}{4})\{[a+b\left(t_1-\frac{1}{\beta}\right)+c\left(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta 2}\right)+d\left(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta 2}-\frac{60}{\beta 3}\right)][\frac{1}{\beta}-\frac{e^{\beta(t_2-t_1)}}{\beta}]+(a-\frac{b}{\alpha}-\frac{60d}{\alpha^3}+\frac{6c}{\alpha^2})(t_2-t_1)+(b-\frac{4c}{\beta}+\frac{36d}{\beta^2})(\frac{t_2^2}{2}-\frac{t_1^2}{2})+(c-\frac{9d}{\beta})(\frac{t_2^3}{3}-\frac{t_1^3}{3})+d(\frac{t_2^4}{4}-\frac{t_1^4}{4})\}+(\Delta_8-\Delta_7)/4[(T-t_2)A+B(\frac{T^2-t_2^2}{2})]-R(T-t_2)[1-A-\frac{B}{2}(T+t_2)]-C[-\{a+b(t_1-\frac{1}{\beta})+c(t_1^2-\frac{4t_1}{\beta}+\frac{6}{\beta^2})+d(t_1^3-\frac{9t_1^2}{\beta}+\frac{36t_1}{\beta^2}-\frac{60}{\beta^3})\}e^{\beta t_1}+[a-\frac{b}{\beta}+\frac{6c}{\beta^2}-\frac{60d}{\beta^3}]]-Cq-C[Aa(t_2-T)+\frac{aB}{2}(t_2^2-T^2)+\frac{Ab}{2}(t_2^2-T^2)+\frac{Bb}{3}(t_2^3-T^3)+\frac{Ac}{3}(t_2^3-T^3)+\frac{Bc}{4}(t_2^4-T^4)+\frac{dA}{4}(t_2^4-T^4)+\frac{Bd}{5}(t_2^5-T^5)]\}$$

..............................(20)

Hence it is possible to get the optimum solution for any particular situation with the help of the above equation (20). Also numerical solution can be obtained with suitable software.

CONCLUSION

In real life, uncertainty plays an important role in formulating a decision making problem of any industry or business company. Due to this uncertainty several parameters may be imprecise. To deal with such problem fuzzy logic is used. In this paper, a two warehouse Fuzzy inventory model is developed considering demand as cubic function of time t, different cost parameters are considered to be fuzzy in nature. Here triangular fuzzy number is used to depict the cost parameters. The deterioration factor is also taken in to consideration here as almost all the products will undergo decay in the course of time due different factors and different preserving facilities in both the warehouses. Also it is assumed that preserving facility of RW is better than in OW. With the help of our formulated system any numerical illustration can be studied with suitable values.

REFERENCES:

[1] Aggarwal, S.P. and Jaggi, C.K. (1995), Ordering policies of deteriorating items under permissible delay in payments, Journal of Operational Research Society (J.O.R.S), 46,658-662.

[2] Benkherouf, L A. (1997), A deterministic order level inventory model for deteriorating items with two storage facilities, International Journal of Production Economics, 48, 167-175.

[3] Balkhi,Z.T. and Benkherouf, L. (2004),On an inventory model for deteriorating items with stock dependent and time varying demand rates, Computers & Operations Research,31,223-240.

[4] Bhunia, A. K. and Maiti M. (1994), A two-warehouse inventory model for a linear trend in demand, Opsearch, 31, 318-329.

[5] Bhunia, A. K. and Mait, M. (1997b), A two warehouses inventory model for deteriorating items with linear trend in demand and shortages, Journal of Operational Research Society, 49, 287-292.

[6] Bellman, E, and Zadeh, L.A (1970).Decesion making in a fuzzy environment.Management Science17 (4):B141-B164

[7] Covert,R.P and Philip,G.P(1973), "An EOQ model for items with Weibull distribution deterioration",AIIE Trans,5(4),323-329

[8] Deb Choudhury.P and Dutta.P(2015), "A Two Warehouse Inventory Model for Deteriorating Items with Cubic Demand, Quadratic Holding Cost and Variable Backlogging Rate", IJAENT,Volume-2 Issue-10, September 2015

[9] Dave, U. (1988), On the EOQ models with two levels of storage, Opsearch, 25.

[10] Donaldson W.A. (1977), Inventory replenishment policy for a linear trend in demand-an analytical solution, Operational Research Quarterly, 28,663-670.

[11] Ghare, P.M and Schrader,G.P (1963) "A model for exponentially decaying inventory", Journal of Industrial Engineering(J.I.E),14,228-243

[12] Goswami, A. and Chaudhuri, K. S. (1992), An economic order quantity model for items with two levels of storage for a linear trend in demand, Journal of Operational Research Society, .43, 157-167.

[13] Haringa.M.A (1995), Effects of inflation and time-value of money of an inventory model on an inventory model with time-dependent demand rate and shortages, E.J.O.R, 81(3), 512-520.

[14] Hartely, R. V. (1976), Operations Research-a managerial emphasis, Goodyear publishing Company, 315-317.

[15] Kar, S., Bhunia, A. K. and Maiti, M. (2001), Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon, Computers and Operations Research, 28 , 1315-1331.

[16] Lee, C. C. and Ma, C. Y. (2000), Optimal inventory policy for deteriorating items with two warehouse and time dependent demands, Production Planning And Control, 7, 689-696.

[17] Mondal, M and Maiti, M.(1999),Inventory of damageable items with variable replenishment rate, stock-dependent demand and some units in hand, Applied Mathematical Modelling,23(1999),pp.799-807.

[18] Mondal, B., Bhunia, A.K. and Maiti, M. (2007), A model on two storage inventory system under stock dependent selling rate incorporating marketing decisions and transportation cost with optimalrelease rule, Tamsui Oxford Journal of Mathematical Sciences, vol-23(3), 243-267.

[19] Mahapatra, N.K. and Maiti, M. (2005), Multi objective inventory models of multi items with quality and stock dependent demand and stochastic deterioration, Advanced Modelling and Optimization.7, 1, 69-84.

[20] Pakkala, T.P.M. and Achary, K.K. (1992), A deterministic inventory model for deteriorating items with two warehouse and finite replenishment rate, European Journal of Operational Research,57, 71-76.

[21] Panda, S, Saha, S. and Basu,M.(2007),An EOQ model with generalized ramp-type demand and Weibull distribution deterioration, Asia Pacific Journal of Operational Research, 24(1),1-17.

[22] Park, K.S (1987) Fuzzy set theoretic interpretation of economic order quantity, IEEE Transactions on systems, Man and Cybernetics,17(6), 1082-1084.

[23] Sana, S, and Chaudhuri, K.S (2008), A deterministic EOQ model with delays in payments and price-discounts offer, E.J.O.R, 184,504-533.

[24] Sharma, K. V. S. (1983), A deterministic inventory model with two levels of storage and anoptimum release rule, Opsearch, 29, 175-180.

[25] Sharma, K. V. S. (1987), A deterministic order-level inventory model for deteriorating items with Two storage facilities, European Journal of Operational Research, 29, 70-72.

[26] Wu, K.S, Ouyang, L.Y.and Yang, C.T. (2006), An optimal Replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging, I.J.P.E, 101-369-384.

[27] Yadav.A.S and Swami.A. (2013),A two-warehouse inventory model for decaying items with exponential demand and variable holding cost, IJIES.

[28] Yadav .R.K and Vats .A.K. (2014), A deteriorating inventory model for quadratic demand and constant holding cost with partial backlogging and inflation, IOSR.

[29] Yang, H.L (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation, European Journal of Operational Research, 157, 344-356.

[30] Yang, H.L (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation, International Journal of Production Economics, 103, 362-370.

[31] Yao J.S and Lee H.M,(1999) Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoidal fuzzy number. Fuzzy sets and Systems,105,311-337.

[32] Y.W.Zhou(2003).A multi-warehouse Inventory model for items with time –varying demand and shortages. Computers and operation Research,vol 30,no 14,pp 2115-2134.

[33] Zadeh,L.A(1965).Fuzzy sets,Information and control,8,338-353.

[34] Zimmermann, H.J (1991).Fuzzy set theory and its applications ,Kluwer Academic Press: Dordrecht.

[35] Zhou, Y.W. and Yang, S.L. (2003), A two-warehouse inventory model for items with stock-level-dependent demand rate, International Journal of Production Economics, 95, 215-228