Determination and Comparative Analysis of PID Controller Parameters for DC Motor Using Cohen and Coon, Ziegler-Nichols and Particle Swarm Optimization Methods

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Abstract— the problem of identification of controller parameters for DC motor has been considered here. Open loop system has been used for Cohen and Coon Method. The proportional, integral and derivative gains are obtained from the process reaction curve. The loop is then closed. Damped Oscillation of Ziegler-Nichols Method is used. This curve, gives the integral and derivative gain from the period of oscillation. The ISE, IAE and IATE are used as fitness function in Particle Swarm Optimization. Then proportional, integral and derivative gains are found out through number of iterations. At last the outputs of three methods are compared.

Keywords— DC motor, PID controller, Cohen and Coon Method, Ziegler Nichols Method and Particle Swarm Optimization, the Integral of Square Error (ISE), Integral of Absolute Error (IAE), and Integral of Time and Absolute Error (ITAE).

INTRODUCTION
In this paper DC Motor of 12 volts has been used. Unit step supply has been given here. The output is the speed of the DC Motor which has been analyzed. To get best results for the DC Motor, PID controller is used. The parameters of the PID controller are set to obtain better results. Hence this paper aims at tuning the PID controller parameters for ideal output results of a DC Motor. Here a comparison of the methods on basis of Performance Evaluation of Swarm Intelligence on Model-based PID Tuning [14] has also been done.

Here tuning of PID controller parameters is based on Cohen Coon Method, Ziegler Nichols Method [6] and Particle Swarm Optimization method [11], [13] for Speed Control of DC Motor. According to previous research tuning of PID controller parameters based on Particle Swarm Optimization are used for governor system of Synchronous Generator [3], for Linear Brushless DC Motor [7], for the Design of Brushless Permanent Magnet Machines [9], for PMDC Motor Drives Controllers [10] and for DC Motor Drives [2]. Again researches show that PID controllers for DC motor have not only been tuned by simple Particle Swarm Optimization but also through Multi-Objective Particle Swarm Optimization [1] and Performance-Dependent Particle Swarm Optimization [12]. Moreover PI Controller [4], Generalized PID Controller [5] and Dual-line PID Controller [8] are used on PSO for Speed Control of DC Motors.

The Cohen and Coon Method and Ziegler-Nichols Method have been used to determine the PID controller parameters. The Particle Swarm Optimization Method is used to optimize the PID controller parameters. Then a comparative study has been done. All these methods are tried and applied in Matlab only.

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**DC MOTOR BLOCK DIAGRAM AND PARAMETERS**

![Block Diagram of DC Motor](image)

Figure1: Block Diagram of DC Motor

Here the parameter that are used for the DC Motor are

- Motor of inertia of the rotor $J = 0.01$ kg $m^2$
- Motor viscous friction constant $B = 0.1$ N.m.s
- Electromotive force constant $K_e = 0.01$V/rad/sec
- Motor torque constant $K_t = 0.01$ N.m/amp
- Electric resistance $R = 1$ ohm
- Electric inductance $L = 0.5$ H

The Motor transfer function used for this paper is

$$\frac{K}{((Js + B)(Ls + R) + K^2)}$$

After putting the Motor parameters in the transfer function, the transfer function that is obtained:

$$\frac{0.01}{0.05^2 + 0.06s + 0.1001}$$
**PROPORTIONAL INTEGRAL AND DERIVATIVE CONTROLLER**

In this paper PID controller has been used. Maintaining the advantages and disadvantages of the PID controller it has been used in the Cohen and Coon, Ziegler-Nichols and Particle Swarm Optimization techniques.

Transfer function of PID controller is

\[
u(t) = K_P e(t) + K_I \int e(t) dt + K_D \frac{d}{dt}e(t)
\]

(3)

**COHEN AND COON METHOD**

The three parameters: static gain ‘K’, dead time ‘t_d’ and time constant ‘\( \tau \)’ describes the system. These parameters are obtained from the process reaction curve. It is easy to estimate the values of the three parameters.

Cohen and Coon approximation for the ‘best’ controller settings using load changes, and various performance criteria, such as one-quarter decay ratio, minimum offset and minimum ISE is as follows:

\[
G_{PRC}(s) = \frac{\overline{y}(s)}{\overline{c}(s)} \approx \frac{K e^{-t_d s}}{\tau s + 1}
\]

(4)

Thus, \( K = \frac{B}{A} \), at steady state.

\( \tau = \frac{B}{A} \), where ‘S’ is the slope of the sigmoid response at the point of inflection.

\( t_d \) = time elapsed until the system responded.

In this paper Cohen and Coon Method is used for parameter identification. Here the process control loop is opened so that no control action (feedback) occurs. Here the gain of the Proportional-Integral controller is lower than that of the ‘Proportional’ controller and the stabilizing effect of the derivative control mode allows the use of higher gains in the PID controller.

The Step value is unity. The scope is simulated for 10 seconds. The process control loop is open looped according to Cohen and Coon Method. The response found is used to find out the static gain ‘K’, the dead time ‘t_d’ and time constant ‘\( \tau \)’.
**ZIEGLER-NICHOLS METHOD**

By using only proportional action and starting with a low gain, the gain is adjusted until the transient response of the closed loop shows a decay ratio of \( \frac{1}{4} \). The reset time and derivative time are based on the period of oscillation, \( P \), which is always greater than the ultimate period \( P_U \).

With the derivative and reset times as formulated, the gain for \( \frac{1}{4} \) decay ratio is again established by transient response tests.

The ratio of the amplitudes of subsequent peaks in the same direction (due to a step change of the disturbance or a step change of the set point in the control loop) is approximately \( \frac{1}{4} \).

\[
\frac{A_2}{A_1} = \frac{1}{4} \quad (5)
\]

The Ziegler-Nichols closed loop method can be applied only to processes having a time delay or having dynamics of order higher than 3.

A closed loop system is taken with DC Motor as the Plant Model. The output of the DC Motor is analyzed and the Proportional controller gain is varied to obtain the \( \frac{1}{4} \) ratio curve.

**PARTICLE SWARM OPTIMIZATION**

The PSO algorithm is simple and easy to implement. The procedures for implementing PSO are as follows:

1. Assume that, in \( d \)-dimensional search space and \( i^{th} \) particle of the swarm can be represented by vector \( X_i = x_{i1}, x_{i2}, x_{i3}, \ldots, x_{id} \).

2. The velocity of the particle is \( V_i = v_{i1}, v_{i2}, v_{i3}, \ldots, v_{id} \), where \( d \) is the dimension of the search space.

3. For each particle, evaluate the fitness function \( f(X_i) \) in \( d \) variable.

4. Also initialize the best visited position of the particle is \( P_{i-best} = p_{i1}, p_{i2}, p_{i3}, \ldots, p_{id} \) and compare fitness evaluation with \( P_{i-best} \). If \( f(X_i) < f(P_{i-best}) \) then \( f(P_{i-best}) = f(X_i) \).

5. Initialize global best position \( P_{g-best} = p_{g1}, p_{g2}, p_{g3}, \ldots, p_{gd} \). Identify the particle in the neighbourhood with the best success so far. If \( f(X_i) < f(P_{g-best}) \) then \( f(P_{g-best}) = f(X_i) \).

6. Position and velocity of the particle is updated by the following equation:

\[
\begin{align*}
\vec{v}_i(t+1) &= \omega \vec{v}_i(t) + c_1 \vec{R}_1 \cdot (p_{best} - \vec{x}_i) + c_2 \vec{R}_2 \cdot (\vec{p}_g - \vec{x}_i) \\
\vec{x}_i(t+1) &= \vec{x}_i(t) + \vec{v}_i(t+1)
\end{align*}
\]

where

\( c_1 \) and \( c_2 \) are positive constant,

\( R_1, R_2 \) are two random variables with uniformly distributed.
ω is the inertia weight which shows the effects of previous velocity vector on the new vector.

An upper bound is placed on velocity in all dimensions $V_{\text{max}}$.

$$\omega = (\omega_{\text{start}} - \omega_{\text{end}}) \times \frac{(\text{MAXITER}-t)}{\text{MAXITER}} + \omega_{\text{end}}$$

where

$\omega_{\text{start}}$ and $\omega_{\text{end}}$ are the initial and final values respectively.

A large inertia weight means that exploration of particle, while small inertia weight favours of exploitation.

t is the current iteration number and

MAXITER is the maximum number of allowed iteration.

7. Go to step 3 until a criterion is match, usually a sufficiently good fitness or maximum number of iteration.

THE FLOW CHART OF PSO PID CONTROL SYSTEM

1. Start
2. Generate initial population
3. Calculate the range of parameters of PID controller
4. Calculate the fitness function (closed loop DC motor).
5. Calculate the pbest of each particle and then the gbest of population
6. Update the velocity and position with pbest and gbest of particles
7. Maximum number of iteration reached?
   - Yes: Stop
   - No: Go to step 3
Specified decay Ratio, Usually $\frac{1}{4}$

Decay Ratio = $\frac{\text{second peak overshoot}}{\text{first peak overshoot}}$

Minimum ISE

$\text{ISE} = \int_{0}^{\infty} |e(t)|^2 \, dt$

Where $e(t)$ = (set point process output)

Minimum IAE

$\text{IAE} = \int_{0}^{\infty} |e(t)| \, dt$

Minimum ITAE

$\text{ITAE} = \int_{0}^{\infty} |e(t)| \, t \, dt$

Table 1: The Performance Criteria

RESULTS

According to theory stated above and the experiments done the results of the methods are given in sequence.

Cohen and Coon Method

The Cohen and Coon Method is an open loop system. So first, the open loop system is been analyzed here. From the analyses the static gain ‘$K$’, the dead time ‘$t_d$’ and time constant ‘$\tau$’ is obtained. These helps in finding the Proportional, Integral and Derivative gain, which is further used for tuning the controller parameters for ideal output of the DC Motor.

The following are the static gain $K=0.1$, dead time $t_d=0.75$ sec and time constant $\tau=1.0625$ sec. Hence the results are obtained, which are given in Table 2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$K_p$</th>
<th>$T_I$</th>
<th>$K_I$</th>
<th>$K_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional</td>
<td>10.3922</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Proportional Integral</td>
<td>7.1863</td>
<td>0.9747</td>
<td>1.0259</td>
<td>-</td>
</tr>
<tr>
<td>Proportional Integral Derivative</td>
<td>10.8180</td>
<td>1.7684</td>
<td>0.5655</td>
<td>0.3072</td>
</tr>
</tbody>
</table>

Table 2: The Gain Values Of P, Pi, And PID Controller Modes.

The PID gain values obtained are used to tune the DC Motor.

The DC Motor is tuned and the output is shown in Figure 3a.

Ziegler-Nichols $\frac{1}{4}$th Decay Ratio Method

The Ziegler-Nichols $\frac{1}{4}$th Decay Ratio method is a closed loop system. So first the closed loop system is been analyzed. Here the proportional gain is continuously changed till the ratio of the first peak and second peak of the output becomes $\frac{1}{4}$. Here the initial proportional gain is assumed as 100 instead of one because for one to 99 there is a large ratio formed. Second Peak Amplitude $A_2$ has a very small gain in the range of one to 99 compared to First Peak Amplitude $A_1$.

The proportional gain obtained after the analysis is used to find out the integral and the derivative gain. These gains obtained are used for tuning the controller parameters for ideal output of the DC Motor.

Hence $K_p=460$

$K_I = \frac{1}{T_I} = \frac{1.5}{P} = \frac{1.5}{0.21} = 7.143$

$K_D = T_D = \frac{P}{6} = \frac{21}{6} = 0.035$

The PID gain values obtained are used to tune the DC Motor.

The DC Motor is tuned and the output of the DC Motor is shown in Figure 3b.
Particle Swarm Optimization

The time integral performance criteria are the objective functions that are used in PSO programming. Hence the objective functions are ISE, IAE and ITAE.

![Figure 3: Output of two tuning methods.](image)

<table>
<thead>
<tr>
<th>Methods</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Swarm Optimization (ISE)</td>
<td>1.73</td>
<td>0.1100</td>
<td>0.1200</td>
</tr>
<tr>
<td>Particle Swarm Optimization (IAE)</td>
<td>1.73</td>
<td>0.31</td>
<td>0.12</td>
</tr>
<tr>
<td>Particle Swarm Optimization (ISE)</td>
<td>0.87</td>
<td>0.1853</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 3: Results Obtained Through PSO (200 Iterations)

The PID gain values obtained from the PSO programming after 200 iterations for ISE, IAE and IATE shown in Table 3 are used to tune the DC Motor.

![Figure 4: Output of PSO method using various fitness functions](image)

Comparative Analysis

The performance analyses are tabulated on the basis of steady state, rise time and peak overshoots.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Steady State</th>
<th>Peak Overshoot</th>
<th>Rise Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohen and Coon</td>
<td>Obtained</td>
<td>No</td>
<td>More than 10 seconds</td>
</tr>
<tr>
<td>Ziegler-Nichols</td>
<td>Obtained</td>
<td>High</td>
<td>Much less than 10 seconds</td>
</tr>
<tr>
<td>Particle Swarm Optimization (ISE)</td>
<td>Not obtained</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

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Particle Swarm Optimization (IAE) Obtained No Approximately 100 seconds
Particle Swarm Optimization (ITAE) Not obtained No

Table 4: Comparison of the Output of the DC Motor for Various Methods, Using the Tuned Controller Parameters.

CONCLUSION
From Table 4 it can be concluded that Cohen and Coon, Ziegler-Nichols, and Particle Swarm Optimization (IAE) can be compared and analyzed on the basis of Steady State, Peak Overshoot and Rise Time.

Firstly comparing Cohen and Coon Method and Ziegler-Nichols Method, it can be concluded that Ziegler-Nichols Method is better than Cohen and Coon Method. Although there is no peak overshoot present in Cohen and Coon Method as high peak overshoot present in Ziegler-Nichols Method, but following are the reasons behind. a) Cohen and Coon Method is an open loop system and do not give any feedback. Due to which the system cannot rectify any of the disturbances sensed by the Plant Model. But the Ziegler-Nichols Method is a closed loop system and hence has the advantage of feedback. b) The rise time of Cohen and Coon Method is much more than Ziegler-Nichols Method.

Secondly the different objective functions of Particle Swarm Optimization Method are compared. The performance of the technique is evaluated by setting its objective function with ISE, IAE and ITAE. Among the three objective functions it can be concluded that IAE is the best. The reason behind it is that when these three objective functions are analyzed for 100 seconds only IAE obtained steady state. Although ISE and ITAE both do not have any peak overshoot but they do not reach the steady state as IAE, which is desirable for the system.

Thirdly comparing Ziegler-Nichols Method and Particle Swarm Optimization Method with IAE as the objective function, it can be concluded that PSO (IAE) Method is better than Z-N Method. Although Ziegler-Nichols Method has much less rise time than PSO (IAE) but Ziegler-Nichols Method has high peak overshoot which is not desirable for any system.

Hence it can be considered that the Particle Swarm optimization with IAE as objective function is the best method. Cohen and Coon Method and Ziegler-Nichols Method are difficult and time consuming processes. Hence soft computing techniques have been widely used to tune the parameters of PID. PID controllers are tuned using soft computing technique which is Particle Swarm optimization.

REFERENCES:


