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DEVELOPMENT OF AN INTEGRATED AND CONVENIENT METHODOLOGY FOR CHECKING LEVELLING SYSTEMS

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Abstract - Each level and its accompanying rods constitute an integral levelling system. The check of a levelling system is essential in order to ensure the credibility of the geodetic applications where is used. A convenient and cost effective methodology for checking levelling systems is presented through this work. The methodology is based on a **low cost innovate comparator** which constitutes the basic tool. Using this **comparator** the levelling systems, which provide accuracy minor or equal to ±1mm/Km, can be checked within 30 minutes. The process, which should be followed as well as the applied statistical tests are analyzed. These tests respond both for the proper operation of the system and the achievement of its nominal accuracy. Additionally the manufacture of the comparator as well as the selection of the used materials is presented. Moreover the calibration methodology of the comparator is described in detail. Finally the results of four levelling systems that were checked are evaluated. It is proved that both tests are necessary as there are cases that a levelling system achieves the nominal accuracy although it doesn't operate properly due to a systematic error. So, useful conclusions are elicited about the convenience and the use of the methodology.

Keywords— digital level, rods, checking, particular comparator, calibration, standard values, measured values, Least Square Method

INTRODUCTION

Digital levels and rods are considered as a unified system for height differences determination and for this reason a contemporary check is suggested [1]. Today the measurements by using such systems are emerged by a way that constitutes "black box" for the most users.

The uncertainty of height difference measurement by using digital level depends mainly on the effect of aging of the CCD camera [2] [3] and the accuracy of the horizontal plane definition by the line of sight.

Also rods have a significant contribution to the levelling result. They are usually deformed because of bad use and maintenance. Some reasons for rods' distraction are the temperature changes (thermal expansion coefficient) [4], blows that cause its distortion, attrition of the rod's base plate (error of the zero) and damage on the bar code. So the measurements, which are taken by the digital process, may have additional errors. Thus some times the provided uncertainty is larger than the nominal one.

Specialized metrological laboratories worldwide use special comparators with interferometers in order to check mainly invar rods. This facility has the ability to shift accurately the rod electronically, as the level is set opposite, at several distances, in order to take readings. There are two types of comparators, for horizontal and vertical setup of the rod. At the horizontal comparators, mirrors are used for the readings acquisition. Such comparator operates in Germany (Unibw) [5], [6]. At the vertical comparators, the check of both the rods and the system is carried out. Such comparator operates at the metrological laboratory of Technical University in Graz (Austria), [7]. Also the Finish Geodetic Institute (FGI) performs automatic calibration of levelling rods by means of vertical comparator from 1996 and systems' calibration from 2002. Similar comparator also operates at Technical University in Ostrava, in Japan (Geographical Survey Institute) and in Slovenia (University of Ljublana) [8] [9].

The cost of this comparator is hundreds of thousands Euros and for this reason they are very rare and they are used mainly for checking high accuracy levelling systems (greater than ±1mm/Km) [5], [6] as it is unprofitable for checking levelling systems, for basic construction works (minor or equal to ±1mm/Km). So, the majority of levelling systems of the worksites are remained unchecked for years.

Thus a low cost and easy to use comparator would be useful in order to help professionals to check their own levelling system. Additionally, the check procedure should be as easy as possible, quick and probably self –applicable. This facility will have major contribution to the reliability of the provided measurements to a plethora of infrastructure works.

Thus, this paper presents a simple and quick checking methodology for levelling systems. The methodology is based on a low cost innovate particular comparator, which simplifies the process.

Initially the comparator is calibrated in order to produce the **standard values** of a number of height differences between particular planar surfaces, which are defined by the steps of its body.

The main idea is to utilize these standard values and the **measured values** of the same height differences, which are provided by the leveling system being checked, in order to apply the check.

CHECKING METHODOLOGY

The main goal of the methodology is the assessment of both the proper operation of a levelling system (accreditation) and its nominal accuracy according to the manufacturer specifications. The proposed methodology is composed by the following steps:

- The rod is put by turns on each step i (i=0 to 11) of the comparator, where ten readings **r** are taken by the level, which is put at 10 meters distance. The mean value r_i of these readings is calculated.
- The ΔH_i between all the successive comparator's steps are calculated $\Delta H_i = r_{i+1} - r_i$.

The linear equation (1) is applied by means of the least squares method as there are formed eleven height differences.

$$\Delta H_i = m \cdot \Delta H_{ist} + b \qquad (1)$$

where ΔH_i are the **measured values** by the leveling system **being checked**

ΔH_{ist} are the **standard values** ,which have been obtained from the calibration of the comparator.

m is the scale of the leveling system, which shows the identification degree between the measured values and the standard values .

b is the constant displacement on the Y axis namely the **systematic error.**

$\hat{\sigma}_o$, is **the standard error** of the adjustment which represents the random error of the leveling system

Thereinafter the appropriate statistical tests are carried out.The first statistical test [10], [11], [12], deals with the proper operation (accreditation) of the leveling system. By using the factors m and b (equation 1) the corrected measurements ($\Delta H'_i$) of the leveling system being checked are calculated using the following equation:

$$\Delta H'_i = \frac{\Delta H_i - b}{m} \qquad (2)$$

So the residuals υ_i are calculated as the differences between the measured values and the corrected values:

$$\upsilon_i = \Delta H'_i - \Delta H_i \qquad (3)$$

The detection of gross or systematic errors is performed by the comparison of the residuals υ_i to their standard errors. So the variance –covariance matrix (V_υ) of the residuals must be calculated in order to obtain the σ_{υ_i} .

If the inequality of the equation 4 is true, then at that specific area of the rod a malfunction is detected, causing incorrect measurements.

$$\upsilon_i \geq z_p \cdot \sigma_{\upsilon_i} \qquad (4)$$

where $z_p = 1.96$ for confidence level ($p = 95\%$)

The second statistical test [10], [11], [12], is essential in order to prove if the leveling system works according to its nominal accuracy as provided by the manufacturer. The following assumptions are examined by using the standard error of the adjustment σ_0.

$$H_o \rightarrow \hat{\sigma}_o \leq \sigma_\theta \cdot \sqrt{\frac{\chi^2_{1-\alpha,r_b}}{r_b}} \quad \text{or} \quad H_1 \rightarrow \hat{\sigma}_0 \geq \sigma_\theta \cdot \sqrt{\frac{\chi^2_{1-\alpha,r_b}}{r_b}} \qquad (5)$$

Where: σ_θ : the standard error of the single measurement of a leveling system as it is provided by the manufacturer.

r_b : the freedom degree

$\chi^2_{1-\alpha,r}$: The value, which is provided by suitable statistical tables of χ^2 distribution for confidence level $p = 1-\alpha$.

So the null hypothesis (H_o), indicates that the estimated standard error of the adjustment ($\hat{\sigma}_o$), is less than or equal to the nominal value (σ_θ). So, it is accepted for a given confidence level. If the alternative hypothesis (H_1) is valid, then the leveling system does not operate according to the manufacturer's specifications.

THE PARTICULAR COMPARATOR

The determination of standard values of some settled height differences requires a particular comparator. After a thorough investigation and study of many parameters, the comparator was manufactured as it is described in the following paragraphs.

Materials as wood (melamine) and glass as well as screws of aluminum for the connection of the wooden surfaces and metal screws for its stabilization are used. The shape of the comparator is drawn as the Greek letter "Π" having twelve steps (fig.1). It is 1.20m in height, 13cm in length and 7cm in width. These dimensions are appropriate in order that a rod be put there. It has special wooden rails clothed with glass, engraved in concrete place. An independent piece of glass, of 8mm in width, is placed like a shelf in the concrete rails in order to put on the rod (fig.2). Thus the structure permits the vertical placement of a rod during the measurements. The dark shadow parts in figures 1 and 2 present the glass surfaces, which cover the timber in order to be frictionless during the shelf placement [13]. The vertical placement of the comparator and its stabilization on a wall was achieved by the simultaneous use of a portable overall level and the use of two total stations. The total stations are placed at perpendicular positions in relation to the comparator in order to align their vertical cross hairs to its edge. Additionally modern digital constructional laser was used. A built-in level is also placed on the comparator’s body, in order to be continuously checked for its verticality. The installation hall of the comparator must ensure the required free space upwards, for the easy rod placement, stability conditions and adequate lighting for capturing the readings.

The step interval decided to be about 10cm as this distance is close to the length of the CCD projection of the code at sighting distance of 3m [5] for some level’s models. Moreover this step size permits a convenient check process as it reduces the measurements and it minimizes the needed time. Thus, twelve steps have been created.

At the right side of the comparator special screws are allocated in order to support the rod and to stabilize it at the vertical position during the measurements.

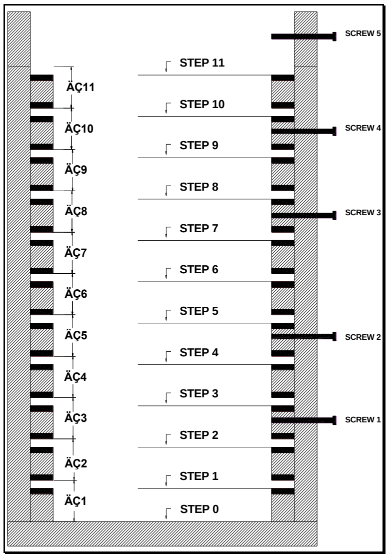

Fig 1. Diagram of the comparator's façade

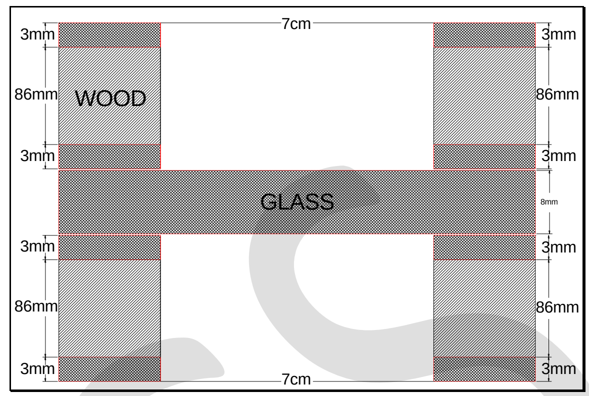

Fig 2. Enlargement of one comparator's step

The comparator, in order to be used for this check, must be unalterable over time and well structured. So, to select the materials that could be used for its manufacturing, two main parameters were examined. These are the thermal expansion of the materials and the bending arrow of each step due to the rod's weight.

The thermal expansion of several materials is given by the equation 6, which calculates the change of a material length dl in relation to the temperature change dT.

$$dl = a \cdot 10^{-6} \cdot l \cdot dT \tag{6}$$

Where: $a \cdot 10^{-6}$: thermal expansion coefficient (grad^{-1})

l : length of the material specimen (m)

dT : temperature change (°C)

Figure 3 presents the influence of the temperature change on one meter of each material namely the thermal expansion coefficient ($a \cdot 10^{-6}$ grad^{-1}) for temperature raise from 2°C to 50°C. Wood (melamine) and Pyrex glass have the minor expansion coefficient $6.6 \cdot 10^{-6}$ and $3.2 \cdot 10^{-6}$ respectively so they have insignificant change of the order of 0.2mm for 50°C temperature change. Also these materials have low cost and for this reason are advantageous and prepossessing.

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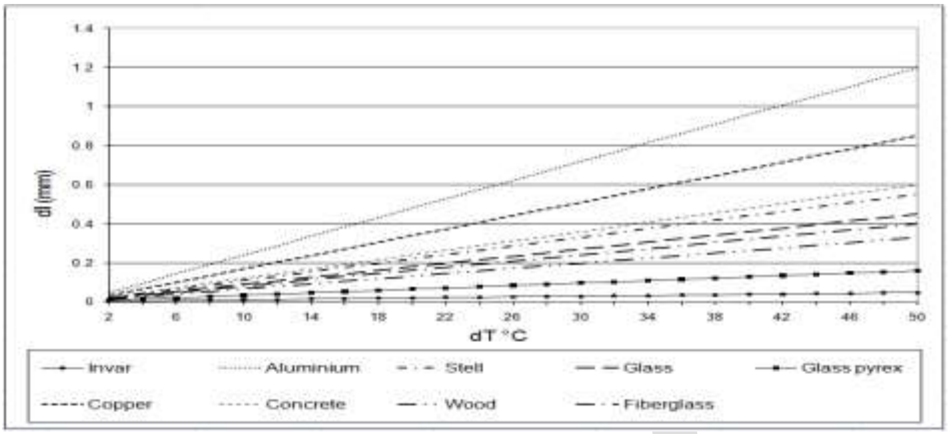

Fig 3. The change of 1m of several materials length in relation to the temperature change

The second parameter, which is examined, is the deformation of a material piece, which will be used as seating surface of the rod. Namely the bending arrow of the seating surface due to the rod's load, which could be considered as distributed load, must be calculated. Equation 7 presents the maximum deformation y_{max} [14].

$$y = \frac{q \cdot x^2}{24EJ} \cdot (l - x)^2, \text{ for } x = \frac{l}{2} \quad \Rightarrow y_{max} = \frac{q \cdot l^4}{384 \cdot E \cdot J} \qquad (7)$$

Where: y_{max} Maximum bending arrow

x : distance from the left edge (m)

l : the length of the material piece (m)

q : distributed Load over the entire surface (kN/m)

E : elasticity modulus (kN/m^2)

J : moment of inertia (m^4)

Figure 4 presents the maximum deformation of several materials caused by rod of about 30Nt ($\approx 3kgr$) as distributed load on a seating surface, which is 130mm in length, 70mm in width and 8mm thick. It's obvious that this deformation is negligible of about 0.2 μm for the most of materials including glass, which additionally is the cheaper.

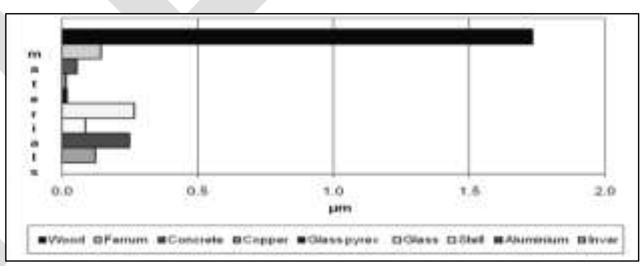

Fig 4. Bending arrow of a seating surface due to the rod's load (distributed load)

The results of the above research show that wood and glass are the most convenient manufacturing materials, as they ensure the maintenance of the comparator's shape and dimensions.

CALIBRATION OF THE COMPARATOR

The calibration of the comparator aims to the accurate calculation of the **standard values** ΔH_{ist} of the eleven height differences, between the twelve steps.

A crucial investigation was carried out in order to calculate the optimum number of readings, which should be taken on each step as well as the appropriate distance between level and rod. Thus an independent experiment was realized as follows. The results are applied both to the calibration procedure and to the check procedure.

Up to 15 readings were taken on an invar rod by using a first order leveling system, from three different positions at distances 5m, 10m and 20m between level and rod.

For each position, 13 separate mean values and their standard errors are calculated taking into consideration each time a different number of measurements (from 3, 4,…to 15).

Figure 5 illustrates the standard error of each mean value on the y axis. The x axis illustrates the number of readings that participate to each calculation. The standard error is reduced, when more readings are used and becomes practically changeless, after the use of ten readings. So the results show that the optimum number is 10 readings. Also the standard error increases as the distance became longer. Therefore was selected the distance of 10m as the difference between 5m and 10m is negligible.

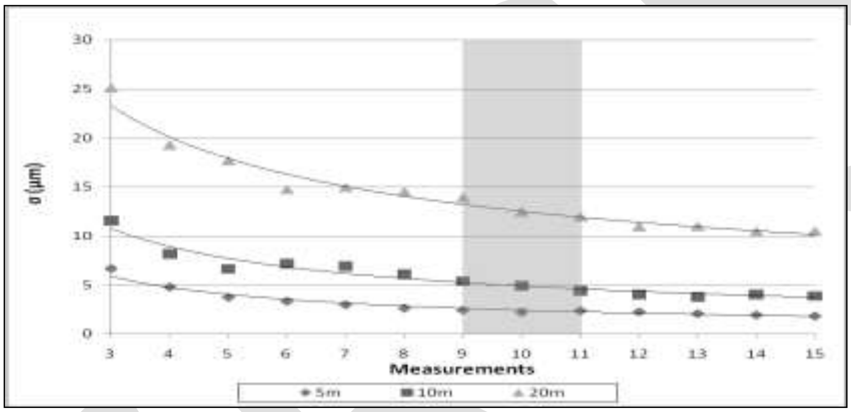

Fig.5 The mean value standard error's change in relation to the number of readings

So the comparator's calibration procedure is as following. Ten repeatable readings are taken on each step. The mean value C_k (k = 0 to 11) of the 10 measurements and the standard deviation σ_{C_k} according to the low of propagation of error, are calculated. Moreover a bilateral statistical test according to distribution F was applied between all the steps in order to examine if the values are of equal weight or not. So exemplarily, the ratio $f = \frac{\sigma^2_{C_k}}{\sigma^2_{C_{k+1}}}$, is created. The null hypothesis is true, namely the readings on steps k and k+1 is of the same uncertainty, when the following equation is fulfilled:

$$F^{1-a/2}_{v_{1-1},v_{2-1}} \leq f \leq F^{a/2}_{v_{1-1},v_{2-1}} \quad (8)$$

Thereafter 55 height differences δH_l are formed (apart from the single ones) as the combination of the 12 steps per two as it is given by the equation 9 and it is illustrated by the figure 6:

$$\delta H_{l,(l=1,\dots,55)} \begin{cases} \delta H_{0-k} = C_0 - C_k, \; k = 2,\dots,11 \\ \delta H_{1-k} = C_1 - C_k, \; k = 3,\dots,11 \\ \dots \\ \delta H_{9-k} = C_9 - C_k, \; k = 11 \end{cases} \quad (9)$$

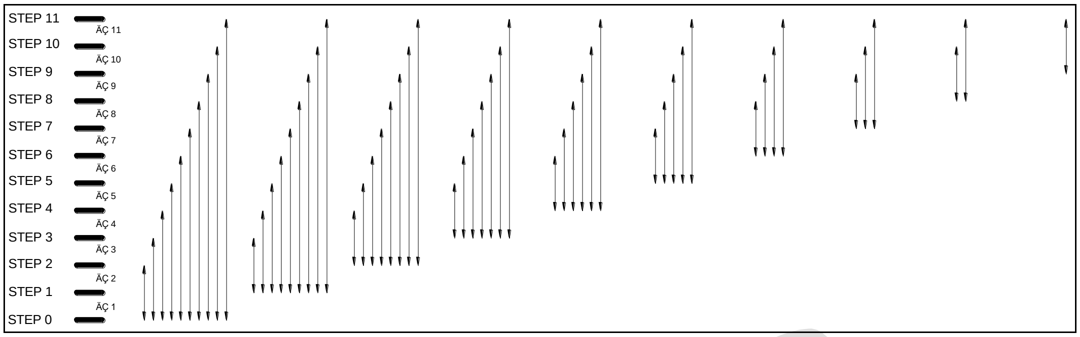

Fig.6 The 55 formed height differences δH_l

In the case that equation 8 is valid for all the steps, then using the 55 equations of the δH_l an equal weight adjustment is carried out in order to calculate the unknowns, which are the single height difference ($\Delta H_{i_{st}}$, i =1, 2,…., 11) between every pair of the comparator's steps $\left(X_1^{11}\right)$. Otherwise, if they are considered unequal weighted and a weight matrix $\left(P_{55}^{55}\right)$ is created by using the standard deviations of every single height difference by applying the law of propagation of error according to equation 10.

$$\sigma_{\delta H_l},_{(l=1,\ldots 55)}\begin{cases}\sigma_{\delta H_{0-k}}=\sqrt{\sigma^2_{c_K}+\sigma^2_{c_0}}, & K=2,\ldots,11\\ \sigma_{\delta H_{1-k}}=\sqrt{\sigma^2_{c_K}+\sigma^2_{c_1}}, & K=3,\ldots,11\\ \ldots & \\ \sigma_{\delta H_{9-k}}=\sqrt{\sigma^2_{c_K}+\sigma^2_{c_9}}, & K=11\end{cases} \quad (10)$$

Then the LSA is used in order to ensure the reliability of the results. So, the standard system $A^T P\hat{x}=A^T P l$ is formed, where A_{11}^{55} is the matrix of the coefficient of the unknowns. The mathematical model of the adjustment is created by the **55 linear height differences equations** is illustrated in equation 11.

$$\begin{aligned}&\Delta H_2+\Delta H_1=\delta H_{0-2}\\&\ldots\\&\Delta H_5+\Delta H_4+\Delta H_3+\Delta H_2+\Delta H_1=\delta H_{0-5}\\&\ldots.\\&\Delta H_{11}+\Delta H_{10}=\delta H_{9-11}\end{aligned} \quad (11)$$

The adjustment has 44 degrees of freedom (55 measured height differences – 11 unknown height differences) and it provides the **standard values** of the eleven height differences ΔH_{ist} (i=1 to 11) and their uncertainties' via the variance - covariance matrix and the standard error $\hat{\sigma}_o$.

Two calibration procedures are carried out in 2011 and in 2015 (~ 4 year span),by using a calibrated first order Leica DNA03 digital level and an invar rod, in order to assess the repeatability of the **standard** values, namely the stability and the maintenance of the comparator over time.

Then the differences $\delta_{\Delta H_{ist}}$ between the two periods are calculated (equation 12), and their standard errors $\sigma_{\delta_{\Delta H_{ist}}}$ (equation 13).

$$\delta_{\Delta H_{ist}}=\Delta H_{i_{st2015}}-\Delta H_{i_{st2011}} \quad (12)$$

$$\sigma_{\delta_{\Delta H_{ist}}}=\sqrt{\sigma^2_{\Delta H_{i_{st2015}}}+\sigma^2_{\Delta H_{i_{st2011}}}} \quad (13)$$

So, the following statistical hypotheses are created for confidence level 95%.

$$H_0 \rightarrow \delta_{\Delta H_i} \leq \sigma_{\delta_{\Delta H_{ist}}} \cdot z_p^{95\%} \quad \text{or} \quad H_1 \rightarrow \delta_{\Delta H_i} > \sigma_{\delta_{\Delta H_{ist}}} \cdot z_p^{95\%} \tag{14}$$

So, for the null hypothesis (H_0), the differences between the two epochs is not statistically significant, while if the alternative hypothesis is true (H_1), the comparator would not be stable.

The differences $\delta_{\Delta H_{ist}}$ fluctuate from 0.01mm to 0.09mm, as the permitted differences $\sigma_{\delta_{\Delta H_{ist}}} \cdot z_p^{95\%}$, fluctuate from±0.04mm to ±0.20mm. So, the standard values of the height differences are successfully passed the test and it is proved that they are repeatable.

The basic concept of the comparison between values, which are measured by an instrument under check, with the corresponding values, which are provided by a higher order instrument, is valid for several metrological checks. The main rule is that the standard instrument should be about 10 times of greater precision than the instruments or the systems, which have to be tested [15].

Therefore according to the results, the comparator is relevant for the check of leveling systems of nominal accuracy of ± 1mm and more. Such a test is recommended to be carried out once a year in order to ensure the standard values and minimizes their determination uncertainty.

EXPERIMENTAL APPLICATION

Four leveling systems of different brands, which provide nominal accuracy ±1.0-1.5 mm/km for 1km double run leveling, were checked [13], [16], by using the comparator, which was established in a laboratory hall.

Pass ✔ Fail ✕	m	σ_m	b (mm)	σ_b (mm)	σ_0 (mm)	P.O	N.A	m	σ_m	b (mm)	σ_b (mm)	σ_0 (mm)	P.O	N.A
	1st leveling system							**2nd leveling system**						
1st Part	1.00049	±0.00012	-0.05	±0.07	±0.11	✔	✔	1.00070	±0.00005	-0.08	±0.03	±0.05	✔	✔
2nd Part	1.00039	±0.00012	0.04	±0.07	±0.09	✔	✔	1.00031	±0.00014	0.17	±0.08	±0.11	✕	✔
3rd Part	1.00029	±0.00015	0.02	±0.09	±0.12	✔	✔	1.00077	±0.00016	-0.11	±0.09	±0.13	✔	✔
4th part	1.00042	±0.00014	0.22	±0.07	±0.09	✕	✔	1.00022	±0.00014	0.07	±0.07	±0.09	✔	✔
	3rd leveling system							**4th leveling system**						
1st Part	0.99981	±0.00022	0.14	±0.15	±0.23	✔	✔	0.99933	0.00018	0.85	±0.12	±0.19	✕	✔
2nd Part	0.99996	±0.00022	0.13	±0.15	±0.23	✔	✔	0.99945	0.00028	0.41	±0.19	±0.29	✔	✔
3rd Part	0.99958	±0.00021	0.66	±0.14	±0.22	✕	✔	1.00000	0.00026	0.00	±0.18	±0.27	✔	✔
4th part	0.99986	±0.00022	0.78	±0.15	±0.24	✕	✔	0.99970	0.00016	0.46	±0.11	±0.17	✕	✔

Table 1. Checking results

All the rods are telescopic, made by fiber – glass and each one could be divided in four parts. The proposed methodology is carried out separately for each part of the rod.

Additionally the statistical tests are applied as given by equations 4 and 5. It is remarkable that, although some of the leveling systems do not meet the conditions of the first statistical test for the proper operation (P.O), their performances are conformed to the nominal accuracy (N.A) according to the specifications, which are set by the manufacturer. Table 1 presents the results.

DISCUSSION

The main advantage of the proposed methodology is the two separate statistical tests that were carried out.

The first test inquires the accordance of the system being checked with the standard system. Thus the system under check is accredited and probable systematic errors are detected.

The second test is about the system's operation in accordance with the manufacturer's nominal accuracy. As it is proved by the experiments a rod's part pass the second test although it fails to the first one.

Therefore only the second test is not enough as cannot detect the systematic errors of the system. These errors aren't also detected via the ISO tests, which only check the internal accuracy of the system [17].

Also the methodology calculates the residuals υ_i for every 10cm segment on the rod's body and so it detects the exact damaged area. It is worth to mention that the methodology can be applied by reducing the magnitude of the step to 5cm or 2cm or even 1cm. This decision induces augmentation of the measurements and also of the time needed to end the procedure.

Moreover by the calculation of the system's scale and the systematic error, it is feasible to determine, the corrected measurements of the system if it has been used to a field work.

The option to check separately each part of the rod is very convenient as each part is short and light enough to be put and to be stabilized at the vertical position on each step of the comparator.

The Least Square Adjustment allows a robust solution providing qualitative and quantitative results with reliability, which could not be achieved by the simple calculation of the differences between the standard and measured values.

Besides, by this procedure the standard error of the single measurement of a system is calculated. In general this information does not be given by the manufacturer as the error of the double running leveling of 1km is registered in the specifications.

The manufacturing of the comparator is very simple and cheap by using common materials. The materials, which were used, are not influenced by the temperature changes or the weight of the rod.

The calibration procedure is reliable and may be repeated in stated time intervals in order to eliminate the uncertainty of the **standard** values of the determined high differences. It is proved by the repeatable calibrations that the differences of the standard values do not exceed the 0.1 mm ±0.05mm so the comparator remains practically invariant over time so, it is relevant for the check of leveling systems of nominal accuracy minor or equal to ± 1mm.

Once the comparator has built and the **standard** values have been calculated the application of the methodology is cost effective and easy to process by any user. This aspect also coincides with the ISO point of view, which is that the procedure is established and organized so as to be self-applicable by any user without the need of any specialized laboratory or personnel, in order to facilitate and to eliminate the check's cost.

CONCLUSIONS

The proposed methodology leads to a fast, easy and total check of levelling systems, both for the proper operation and the nominal accuracy. This is achieved by:

- The manufacturing of the particular comparator.
- The calibration of the comparator by using a first order invar levelling system in order to calculate the **standard** values of the height differences between the comparator's steps.
- The acquisition of the **measured** values of the same height differences by the levelling system being checked.
- The elaboration of the standard and measured values by means of a least square adjustment.
- The two statistical tests, which answer for the proper operation and the nominal accuracy of the levelling system being checked.

The diagram of the figure 7 illustrates the main points of the checking methodology.

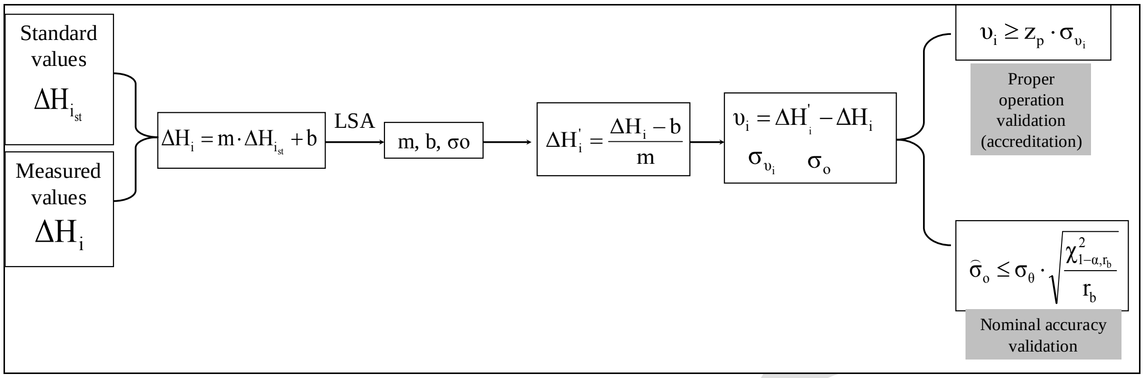

Fig.7 The main stages of the checking methodology.

The process for the check of a system with three meters length rod lasts about 30 minutes.

Consequently this simple comparator consists a cost effective facility and it is worth to be used for checking ordinary levelling systems, as the use of a high performance invar comparator is unpractical and valueless for construction levelling systems of nominal accuracy minor or equal to ± 1mm/km.

The previous mentioned advantages make the proposed methodology efficient, easy and convenient to be used.

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