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Message from IJERGS

This is the First Issue of the Eight Volume of International Journal of Engineering Research and General Science. A total of 15 research articles are published and we sincerely hope that each one of these provides some significant stimulation to a reasonable segment of our community of readers.

In this issue, we have focused mainly on the Young innovative Ideas. We also welcome more research oriented ideas in our upcoming Issues.

Author's response for this issue was really inspiring for us. We received many papers from many countries in this issue but our technical team and editor members accepted very less number of research papers for the publication. We have provided editors feedback for every rejected as well as accepted paper so that authors can work out in the weakness more and we shall accept the paper in near future.

Our team have done good job however, this issue may possibly have some drawbacks, and therefore, constructive suggestions for further improvement shall be warmly welcomed.

IJERGS Team,

International Journal of Engineering Research and General Science

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Adsorption of phosphate from aqueous solution by marble dust

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Abstract— A simple and economic experimental sorption procedure is developed for efficiency removal of phosphates from aqueous solution using experiments batch system. Many experiments were conducted to use marble in powder form which collected from the workshop was used to adsorption phosphate PO_4^{3-} from aqueous solution. Equilibrium concentration has been measured using distribution coefficient to evaluate the removal efficiency. The PO_4^{3-} adsorption experiments were carried out to analyze the influence of governing factors such as pH of the solution, mass of marble dust MD, initial PO_4^{3-} concentration, temperature and contact time, on the adsorption efficiency of PO_4^{3-}. The equilibrium test showed that, the sorption efficiency depended on the pH of the solution. The adsorbed of PO_4^{3-} increased with range (pH 5.0 - 6.5). It was found that the maximum adsorption capacity of MD for removing of PO_4^{3-} reached to 13.68 mg g^{-1} and 13.57 mg g^{-1} at pH 6.0 and 6.5 respectively.

Keywords— marble, phosphate, adsorption, removal, water, pH value, batch system

introduction

Phosphorus is the most abundant element on the surface of the earth, and its common compounds are abundant in phosphates, which exist in water system naturally by dissolving out of igneous rock, anthropogenic activities and the environmental practices of industries and agricultural practices through the use of large quantities in fertilizers [1], detergents [2] and some other industries [3]. PO_4^{3-} plays an important role in biochemical processes as well as enriching surface water [4]. The excess of phosphate into the water bodies causes a significant low of oxygen and excessive growth of algae [5] which causes of environmental system for plants that need oxygen. Some studies confirmed that the exposure to high concentration of PO_4^{3-} cause chronic kidney disease and cardiovascular [6-8].Various technologies are employed for removing PO_4^{3-} from polluted waters such as HFeO [9] Mg-laden biochar synthesized at different temperature [10], lanthanum-modified zeolites [11]. Many previous studies used low-cost material which requires little processing, ,abundant in nature, or a by-product or waste material from another industry such as shale, sandstone, and laterite [12]. Electrochemical removal is one of the most procedural for removing PO_4^{3-} by using Electrocoagulation as a green technology [13, 14].

Various Mesoporous sorbent materials, including metal-coordinated amino-functionalized silicas, ammonium-functionalized silicas [15] and ,6-amine–functionalized copper ferrite [16] have a strong affinity for removing phosphate. Oxide as Fe_3O_4, nanocomposite of titanium oxides and magnetite core/zirconia is expensive, available and chemically stable over a wide pH range. Many previous studies [17-20] confirmed that the oxides have high sorption affinity toward phosphate. The use of natural sorbent material of the removal contaminated substance such as activated carbon, clay. Zeolite, vegetal cords, carp scales, [21-24] which give promises results with low cost technologies. The use of the those natural or recycled materials in the removal of PO_4^{3-} such as seashell, biochar, proteins, cotton stalk, steel byproduct [25-28] provide high adsorption capacity without additional cost. One of the sorbent materials which give high capacity in the removal of PO_4^{3-} from aqueous is marble dust MD as a natural material, which produced from crashing and cutting of the founded marble block, where the main component is calcium which has good affinity for bonding with PO_4^{3-}ions as conception to form of amorphous calcium phosphate $Ca_2(PO_4)_2$, dibasic calcium phosphate $CaHPO_4 .2H_2O$

, and $Ca_5(PO_4)_3OH$ [29]. A simplified scheme of the MD process (chemical sorption) based on the activity of Ca^{2+} is presented in fig.1

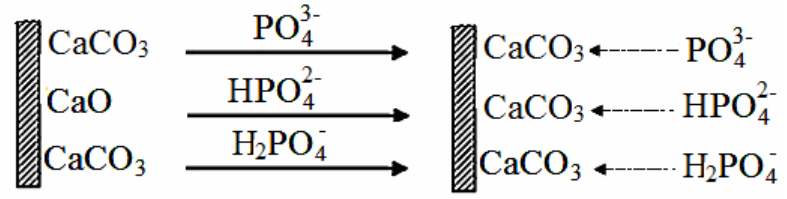

Fig.1. Scheme of the sorption of PO_4^{3-} on the surface of MD

This research reports the use of marble dust as natural sorbent materials, inexpensive and easy to find to remove $PO_4^{3-}$from aqueous solution by forming insoluble metal phosphate.

EXPERIMENTAL PROCEDURES

Materials

In this experimental work, the following chemicals were used: sodium dihydrogen phosphate KH_2PO_4 p.a Aldrich (Munich, Germany); HNO_3 p.a sigma; NaOH p.a Merck; Ammonium molybdate , ascorbic acid ; Stannous chloride $SnCl_2$ p.a Merck.

Phosphate

A stock solution of PO_4^{3-} (1000 mgL^{-1}) was prepared by dissolving 1.43g of sodium dihydrogen phosphate KH_2PO_4 in 1L: volumetric flask with deionized water.

Method for the determination

The concentration of PO_4^{3-} before and after adsorption was analyzed with ammonium molybdate method by using a CO7500 Colorimeter (WPA UK)

Physical and chemical properties of MD.

The marble powder used in this study was collected from marble workshop considered as refused material. The chemical and physical properties are listed in Table 1.

Table 1. Physical and Chemical Properties of Marble Powder

Chemical Composition		**Physical Properties**	
Test	**Mass %**	**Test**	**Unit**
Loss on ignition	39.05	Moisture 0.036	%
Calcium Carbonate	98.61	Particle Size <300	micron
Magnesium Carbonate	0.59	Density 1.42	g/cm^3
Calcium Oxide	55.16		
Silicon Oxide	1.09		
Aluminum Oxide	0.69		

The MD was used in this experimental work without any further preparation.

Method of sorption process

The phosphate sorption process was investigated in the batch system, a vessel used was 500mL glass beaker holding of the PO_4^{3-} solution, with a range (0.5 mgL^{-1} – 15 mgL^{-1}) concentration of 100 ml of solution, a shaker speed used to distribute the MD into solution at 25 C°,30 C°, and 40 C° was 120 rpm, the pH of solutions were conducted within range (pH 3.0 to pH 8.0), the mass of MD was varied from (0.5g to 5.0g) and the contact time was constant (90min).

The following equation was used to evaluate the sorption capacity:

$$q = \frac{C_i - C_f}{m} V$$

Where:

q : sorption capacity, mg/g.

C_i : initial PO_4^{3-} concentration, mgL^{-1}

C_f : final PO_4^{3-} concentration, mgL^{-1}

m : mass of MD, g.

V : volume of solution, L.

RESULTS AND DISCUSSION

Effect of pH

The effect of pH value of aqueous solution contaminated with PO_4^{3-} on the adsorption capacity was studied. The conditions of the procedure were: pH value was varied pH 3.0 to pH 8.0, mass of MD (m=1.0 g), concentration of PO_4^{3-} (15 mgL^{-1}), temperature (room temperature), volume of solution (V= 100 mL), shaker speed (120 rpm), and contact time (90 min) was constant.. The efficiency of phosphate adsorbed on MD is calculated for each pH value. The mass of PO_4^{3-} removed from aqueous solution is presented in fig.2.

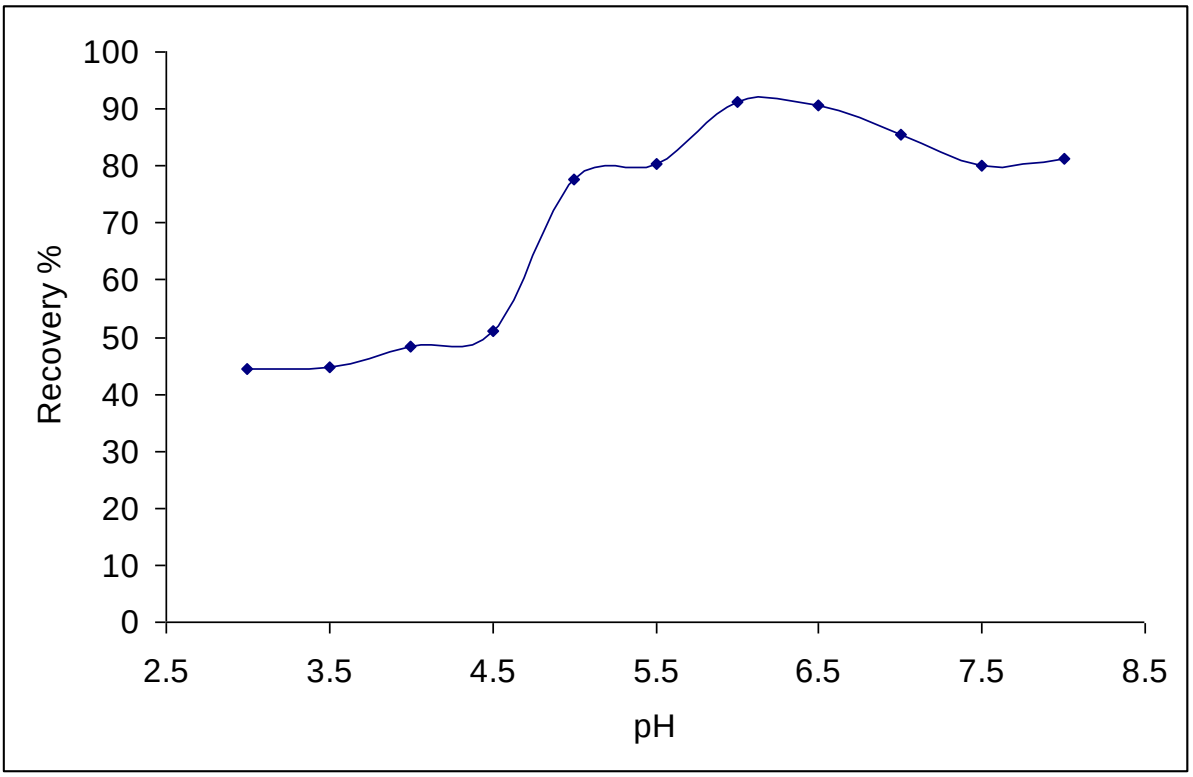

Fig.2 The effect of pH on removing PO_4^{3-} by using MD

The fig.2 showed that the adsorption of PO_4^{3-} by using MD is affected by pH value, the PO_4^{3-} is highly sensitive to the change in pH. At low pH value 3.0 to 5.0 (acidic media) the removal capacity up to 50% , while at pH 4.5-6.0 up to 80% .The optimal pH for MD is 5.5 and 6.5, at this pH value the MD has excellent efficient reached to 91.0% . At higher pH value the Ca-MD is consumed in the form of hydroxides $Ca(OH)_2$

Effect of MD mass

Elucidate the effect of the mass of MD on adsorption capacity was studied and presented in fig.3, the procedures were carried out following the standard experimental, pH of the solution was constant 6.0, concentration of PO_4^{3-} (0.5, 1.0, 5.0, 10, 15 mgL^{-1}), volume of solution (V=100 mL), temperature (room temperature), time of contact (90 min), shaker speed(120 rpm) and mass of MD was varied between (0.5g and 5.0 g). For each mass of MD the capacity of PO_4^{3-} adsorbed (in %) and bonded to the MD is calculated and presented.

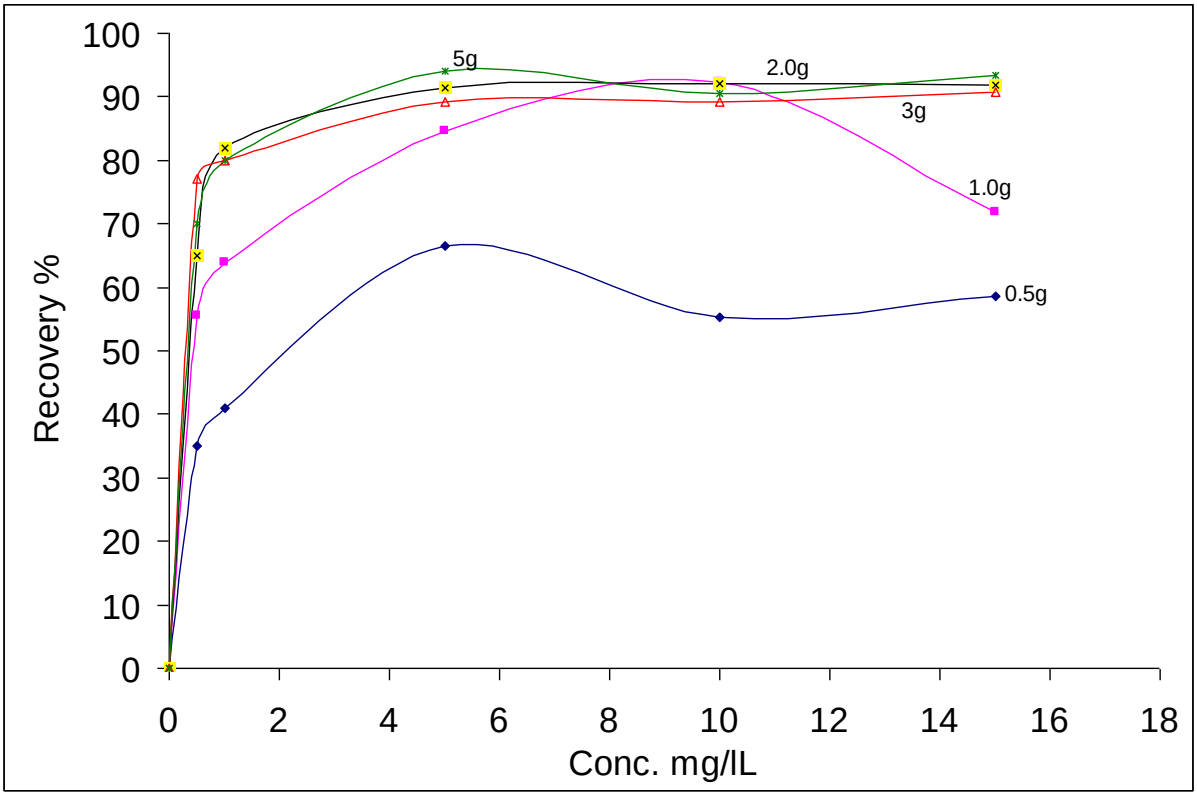

Fig. 3 Adsorption capacity of MD for PO_4^{3-} at various masses

Fig.3 shows the results of influence of MD mass on adsorption of PO_4^{3-}. The PO_4^{3-} adsorption efficiency increased with the mass of MD increase , 58% at 0.5g of MD , 93.5% at 5.0 g, while the adsorbed capacity was approximately the same it were, 91.5%, 89% and 94% at 2.0g, 3.0g, 5.0g respectively. In this case the concentration of PO_4^{3-} was 10 mg L^{-1}.

Effect of temperature

The influence of temperature on the adsorption of PO_4^{3-} was studied at temperature rang (20 °C to 35 °C) . The results are shown in fig.4. Removal of PO_4^{-3} was conducted using the following condition: the concentration of PO_4^{3-} (10 mgL^{-1}), mass of MD (m= 2.0g), pH of the solution (pH 6.0), volume of solution (V=100 mL), shaker speed (120 rpm) and contact time (90 min).

The result shows that the adsorption capacity of MD with respect to PO_4^{-3} was good recoveries at 25°C and 30 °C , while the adsorption capacity shows acceptable affinity at 35 °C due to dissolving of calcium carbonate and calcium phosphate with increase of temperature, where the solubility of PO_4^{3-} is dependent of pH of solution.

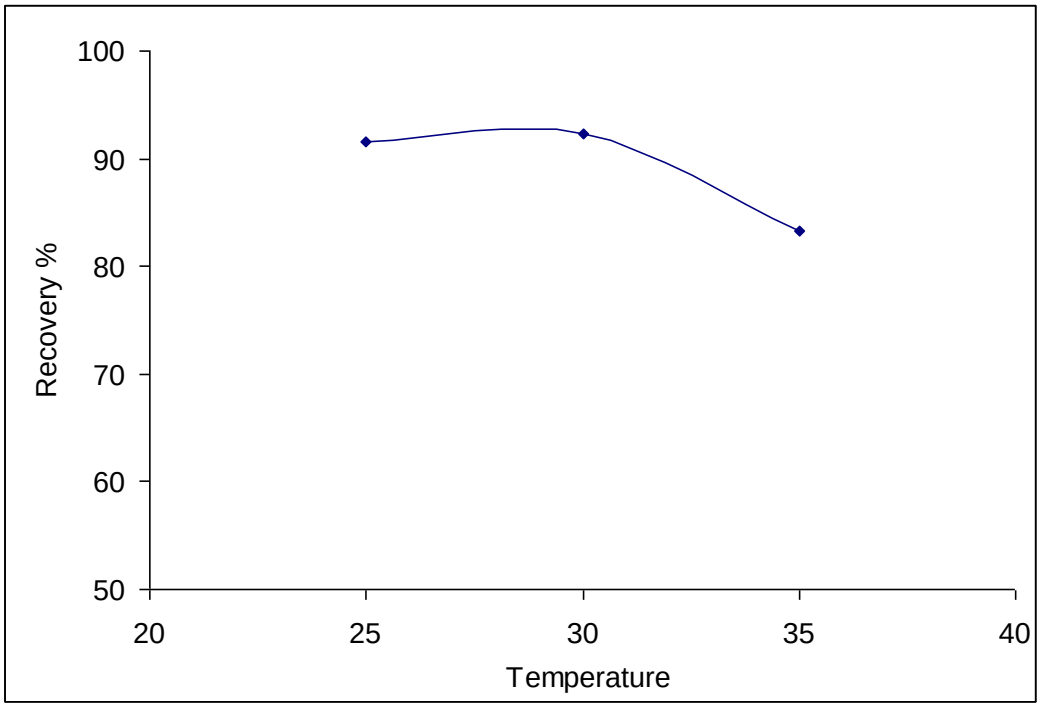

Fig.4. Effect of temperature on removal of PO_4^{3-}

Interference: effect of Carbonate

The carbonates as the common ions present in water, and MD as sorbent material are strongly affected by pH value. The results of MD tests for efficiency adsorbed PO_4^{3-} and carrying carbonate concentration (at pH=6 converted to bicarbonate) in the solution was studied and presented in fig.5.

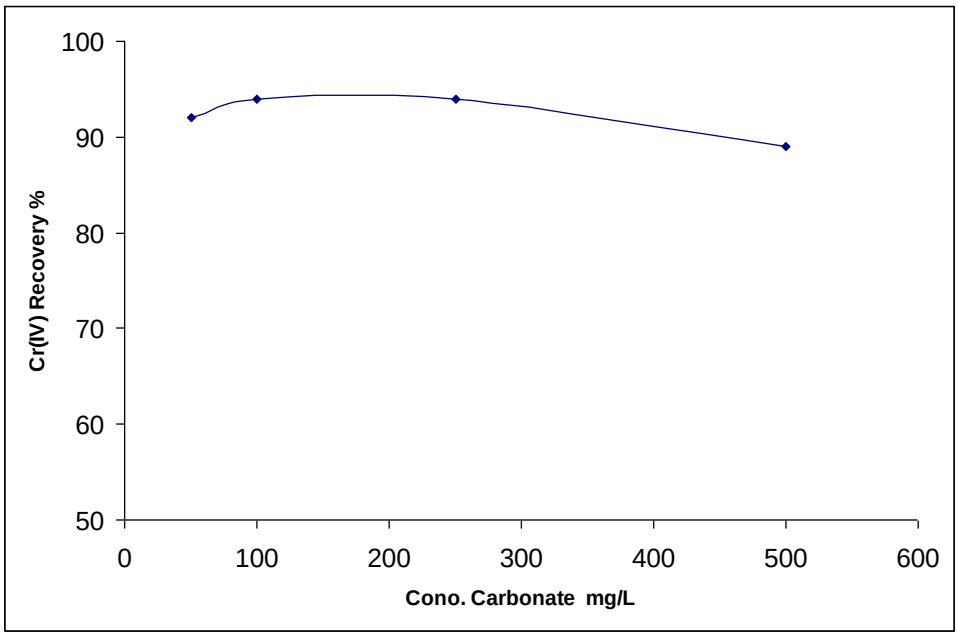

Fig5. The effect of Carbonate on adsorption process.

The conditions procedures were: mass of MD (m=2.0 g), volume of solution (V=100 mL), concentration of PO_4^{3-} (10 mgL^{-1}), pH value (pH= 6.0), contact time (90 min), shaker speed (120 rpm), temperature (room temperature), and concentration of carbonate was

varied (50 mgL^{-1}, 100 mgL^{-1}, 250 mgL^{-1} and 500 mgL^{-1}). The efficiency of MD to remove PO_4^{3-} from aqueous solution was measured and the result showed that the 50 mgL^{-1} , 100 mg L^{-1} and 300 mg L^{-1} of carbonate not have any effect on capacity of adsorbed PO_4^{3-}, where the 500 mgL^{-1} of carbonate will have slightly effected on adsorption capacity.

Applications simple method to standard solution and real water samples

Standard samples

Four standard samples of deionized water were spiked with addition of different concentration of PO_4^{3-} , the experiment was based through the optimum procedure, mass of MD (m=2.0 g), volume of samples (V=100 mL), pH of water (pH = 6.0), contact time (90 min), and shaker speed (120 rpm).The spiked range of deionized water was 0.5 mg L^{-1}, 1.0 mgL^{-1}, 5.0 mgL^{-1} and 10.0 mgL^{-1}). The results of samples analysis were spiked with different concentration of $PO_4^{3-}$are presented in Table 2.

Table 2. Results of Adsorption recovery by MD in spiked standard solution.

Standard solution	PO_4^{3-} content , standard addition mg L^{-1}	Removed mg L^{-1}	Recovery %
1	0.5	0. 45±0.004	91.45
2	1.0	0.967±0.03	96.7
3	5.0	4.56±0.052	91.2
4	10	9.31±013	93.1

The results in table 2 showed that, the good recoveries of 91.5%, 96.7% , 91.2% and 93.1 % were obtained and the relative standard deviation (RSD) was 0.8, 2.98 , 1.14, and 1.4 for 0.5, 1.0, 5.0 and 10 mgL^{-1} respectively.

Real water samples

The Results of two real water samples were spiked with different concentration of PO_4^{3-} presented in the table 3. The concentration of carbonate and bicarbonate of two samples were analyzed before adsorption procedures (nil CO_3^{2-} , 34.1 mgL^{-1} HCO_3^{2-} , nil CO_3^{2-} , 170 HCO_3^{-}) for sample 1 and 2 respectively. The procedure of adsorption of PO_4^{3-} based through the optimum procedure, mass of MD (m=2.0 g), volume of water samples (V=100 mL), pH of water (pH = 6.0), contact time (90 min), and shaker speed (120 rpm).

Table 3. Results of adsorption of PO_4^{3-} using MD in real water samples spiked with addition different concentration of PO_4^{3-}.

Sample No.	PO_4^{3-} content , standard addition mg L^{-1}	Removed mg L^{-1}	Recovery %
1	5.0	4.72±0.05	94.4
1	10	9.20±0.12	92.0
2	1.0	0.93±0.017	93.1
2	10	9.66±0.23	96.6

The results presented in table 3, showed that good recoveries were found in the two samples (94.4%, 92.0%) and (93.1%, 96.6%) while RSD were (1.1% ,1.31%) and (1.8% , 3.2%) respectively.

CONCLUSION

Adsorption of PO_4^3 on natural material was studied using marble dust as sorbent material. The pH of solution had a significant effect on the adsorption of PO_4^{3-}. When the mass of MD was 2.0 g , 96% removal efficiency was achieved. The reaction occurred in mass of MD scale and the reaction efficiency increased significantly from 0.5 g to 1.0 g, while the reaction between MD and PO_4^{3-} approximately had constant removal efficiency with increasing of mass of MD from 1.0 g to 5.0 g at pH value 6.0, the adsorption of PO_4^{3-} was favorable, where the effect of the presence of carbonate ions in the sample had no effect on the adsorption of PO_4^{3-}. This study indicated that the MD can be used as natural material which has good capacity in bonding of PO_4^{3-} in water samples.

REFERENCES:

1. Wandruszka, R. V., Phosphor retention in calcareous soil and the effect of organic matter on its mobility, *Geochem. Trans*. 6(7), (2006) 1-8
2. Shyla, B., Mahadevaiah, and Nagendrappa,G., A simple spectrophotometric method for the determination of phosphate in soil, detergents, water, bone and food samples through the formation of phosphomolybdate complex followed by its reduction with thiourea, *Spectrochim. Acta A* 78(1), (2011) 497-502.
3. Oladegi, S., Adelowa, F., and Odelade, K., Evaluation of level in water samples (Ogbomoso Rivers) using UV-Visible spectrophotometric method, *J. Scien. Rese. Envion. Sc*i. 4(4), (2016) 102-108.
4. Sawyer,C. N., Perry,M. L., and Gene, P. F., Chemistry for Environmental Engineering and Science.5th Ed, McGraw- Hill Publishing Company Ltd: 677 (2002).
5. Abowei, J. F. N., Davies,O. A., and Eli, A., Physicochemistry, morphology and abundance of fin fish of Nkoro River, Niger Delta, Nigeria. *Int. J. Pharm. Biosci*., 6(2), (2010) 1-11.
6. Ritz, E., Hahn, K., Ketteler, M., Kuhlmann, M. K., and Mann, J., Phosphate additives in food – a health risk, *Dtsch. Arz. Inter.*109, (2012) 49-55.
7. Foley, RN., Collins, AJ., Herzog, CA., Ishani, A., and Kalra, PA. Serum phosphorus levels associate with coronary atherosclerosis in young adults, *J. Am Soc Nephrol*, 20 (2), (2009) 397-404.
8. Tonelli, M., Sacks, F., Pfeffer, M., Gao, Z., and G. Curhan, Relation between serum phosphate level and cardiovascular event rate in people with coronary disease, *Circulation*, 112, (2005) 2627–2633
9. Ramirez, A., Giraldo, S., Garcia- Nunez, J., Florez, E., and Acelas, N., phosphate removal from water using a hybrid material in fixed-bed column, *J. Watert. Proc. Eng.* 26, (2018) 131-137.
10. Jiang, D., Chu, B., Amano, Y., and Machida, Mremoval and recovery of phosphate from water by Mg-laden biochar: Batch andcolumn studies, *Colloids Surf., A*, 5584, (2018) 29-437.
11. J. Goscianska, M. Ptaszkowask, M. Frankowski, M. Franus, R. Panek, and W. Franus, Removal of phosphate from water by lanthanum-modified zeolites obtained from fly ash, *J. Colloid Interface Sci.,* 513, (2018) 72-81.
12. Mallet, M., Coulibaly, L., Abdelmoula, M., and Ruby, C., Phosphate removal from water by natural occurring shale, sandstone, and laterite: The role of iron oxides and of soluble species, *C.R. Geosci.,* 351(1), (2019) 37-47
13. Hashim, K. S., AlKaddar, R.. Jasim, N., Shaw, A., Phipps, D., Kot, P., O. Pedrola, M., Alattabi, A., Abdulredha,M.,and Alawsh, R., Electrocoagulation as green technology for phosphate removal from river water, *Sep. Purif. Technol.*210, (2019) 135-144.

14. Hashima,K. S. Idowu,I., Jasim,N., Al Khaddar, R., Shaw, A., Phipps, D., Kot, P., Pedrolaa, M., Alattabi, A., Abdulredha ,M., Alwash, R., Teng, K.H., Joshi, K.H., and Aljefery M. H., Removal of phosphate from River water using a new baffle plates electrochemical reactor, *Methods X*, 5, (2018) 1413-1418.
15. Zhang, J., Shen, Z., Mei, Z., Li, S., and Wang, W., Removal of phosphate by Fe-coordinated amino-functionalized 3D Mesoporous silicates hybride materials, J. Environ. Sci., 23(2) (2011) 199-205.
16. Gu, W., Xing, M., Fang, W., and Wu, D., Removal of phosphate from water by amine-functionalized copper ferrite chelated with La(III) *Sci. Total Environ*. 619-620 (1), (2018). 42-48.
17. Jiang, D., Amano, Y., and Machida, M., Removal and recovery of phosphate from water by a magnetic Fe_3O_4@ASC adsorbent, *J. Environ. Chem. Eng*.5 (5), (2017). 4229-4238.
18. Nie, G., Wu L., Du, Y., Wang, H., Yanhua ,X., Ding, Z., and Liu, Z.,). Efficient removal of phosphate by a millimeter-sized nanocomposite of titanium oxides encapsulated in positively charged polymer, *Chem. Eng. J*. 360, (2019) 1128-1136.
19. Wang, Z., Xing, M. Fang, and Wu, W. D., One-step synthesis of magnetite core/zirconia shell nanocomposite for high efficiency removal of phosphate from water *Appl. Surf. Sci*, 366, (2016) 67-77.
20. Gan, L., Lu, Z., Cao, D., and Chen, Z., Effect of cetyltrimethylammonium bromide on the morohojogy of green synthesized Fe_3O_4 nanparticles used to remove phosphate, *Mater. Sci. Eng., C.* 82, (2018) 41-45.
21. Pavonia, B., Drusiana, D., Giacomettia, A., and Zanetteb, M., Assessment of organic chlorinated compound removal from aqueous matrices by adsorption on activated carbon, *water Res*. 40, (2006) 3571-3579.
22. Buchanan, W., Roddick, F., and Porter, N., Removal of VUV pre-treated natural organic matter by biologically activated carbon columns, *Water Res.*42, (2008) 3335-3342.
23. Cherifi, H., Hanini, S., and Bentahar, F., Adsorption of phenol from wastewater using vegetal cords as new adsorbent, *Desalination*, 244, (2009) 177-187.
24. Bajic, Z., Djokic, V., Velickvic, Z., Vuruna, M., Ristic, M., Benissa, N., and Marinkovich, A., Equilibrium kinetic and thermodynamic studies on removal of Cd(II), Pb(II) and As(V) from wastewater using carb (Cyprinus Carpio) scales, *Dig. J. Nanomter. Bios*.8, (2013) 1581-1590.
25. Christoph, J., Lina, P., Krenz, M.,and Pleissner, D., Is seashell powder suitable for phosphate recovery from fermentation broth, *New Biotechnology*, 49, (2018) 43-47
26. Jiang, D., Chu, B., Amano,Y., and Machida, M., Removal and recvery of phosphate from water by Mg-Laden biochar: Batch and column studies, *Colloids Surf.,A*, (2018) 558, 429-437.
27. Kiteshwarna, K. Pokhrel, N., Hussein, F., Antony, E., and Mayer, B., Phosphate removal and recovery using immobilized phosphate binding proteins, *water Res. X*, 1, (2018)1-9.
28. Sellner, B. M., Hua, G., and Ahiablame, L. M., Fixed bed column evaluation of phosphate adsorption and recovery from aqueous solution using recycled stell byproducts, *J. Environ. Manage.* 233, (2019) 595-602.
29. Wandruszka, R. V., Phosphorus retention in calcareous soils and the effect of organic matter on its mobility, *Geochemical transaction*, 7 (6), (2006) 1-8.

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When is the Optimal Time to Retire?

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Abstract- This paper investigates the optimal age of retirement by considering many factors. The main factor is the system in which the employee is working. We use the CALPERS (California Public Employees Retirement System) data in our study. The method and mathematical tools we use, however, is not restricted to CALPERS system and can be generalized to any other systems. In addition, we also examine the longevity risk factor that affects the optimal retirement age. Overall, we find the optimal age of retirement is determined by the benefit an employee receives after retirement and the longevity of the employee.

Keywords: CALPERS, Retirement age, benefit factor, longevity

Introduction

As people live longer, sufficient and efficient provisions for retirement are more important. A good retirement plan could save one from outliving his or her retirement wealth, or not able to collect sufficient benefits before death. In this paper, we calculate optimal age of retirement based on mathematical models. We use the data from CALPERS (California Public Employment Retirement System). The results are particularly important to CALPERS employees. While the specific results of this study are geared to CALPERS employees, the method and the mathematical tools can be easily generalized to any retirement system. We first derive the basic equation between retirement age, number of years of service, and benefits which is measured by percentage of gross salary. However, retirement depends not only on the amount one would get after retirement but also on the longevity of the person. Once longevity comes into the picture, risk factor comes into the corresponding mathematical model. To address this issue, we collect information on longevity study of employees. Then we derive a mathematical function between a person's retirement age and the number of years they live after retirement. These functions will then be combined to get an integrated function which yields the optimal age of retirement. On a plot, the optimal age of retirement is the intersection of the two functions. The problem becomes quite

complex as the retirement benefits vary on the number of years of service. This research considers all the important and relevant information regarding retirement and longevity of the employee to calculate the optimal age of retirement.

CALPERS provides a table of benefit factors for the CALPERS employees with various years of service. This data is shown for the case of 2% at 55 (CALPERS, 2002). In addition, CALPERS also provides the percentage gross salary versus retirement age. These benefit factors are used to calculate the percentage of gross salary of an employee which is essentially the product of benefit factors and number of years of service. It is to be noted that the benefit factors vary with the age of retirement. The optimal age of retirement is determined by many factors. The main factor is the system in which the employee is working. In addition, there are other factors such as health, psychological and sociological factors (Putcha and Sloboda, 2015; Putcha and Sloboda, 2013). Two approaches can be used to solve the problem of optimal retirement age. Both approaches are discussed and the results are reported in this paper. Overall, we find the optimal age of retirement is determined by the benefit an employee receives after retirement and the longevity of the employee.

2. Data Collection

We obtain our primary data from CALPERS where benefit factors for different years of service are tabulated. CALPERS change retirement plans from time to time. Table 1 is CALPERS retirement plan released in 2002. This is also known as "the 2% at 55 formula" because the benefit factor is 2% when employees retire at age of 55. In addition, CALPERS also provides the percentage gross salary versus retirement age. The data is shown in Table 2.

Table 1: Retirement Age and Benefit Factor (CALPERS, 2002)

Retirement Age	Benefit Factor (%)
50	1.100
51	1.280
52	1.460
53	1.640
54	1.820
55	2.000
56	2.064
57	2.126
58	2.188
59	2.250
60	2.314
61	2.376
62	2.438
63+	2.500

Figure 1 is a graphic representation of Table 1.

Figure 1: Retirement Age and Benefit Factor (CALPERS, 2002)

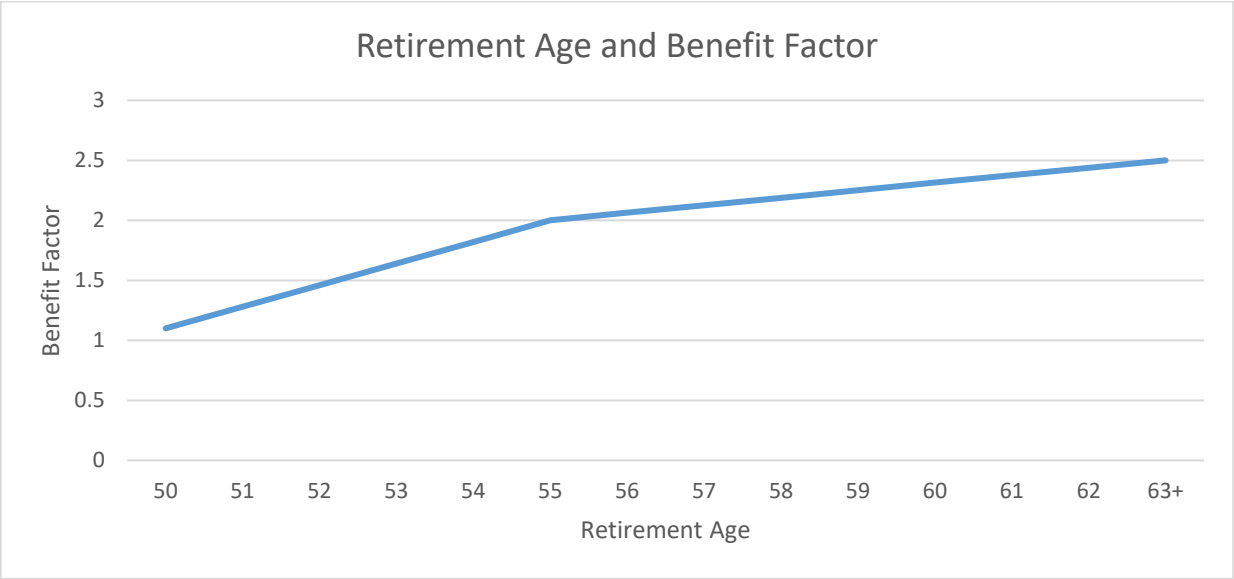


Table 2: Percentage of Gross salary for various years of service and retirement age (CALPERS, 2002)

	Years of service						
	5 years of service	10 years of service	15 years of service	20 years of service	25 years of service	30 years of service	35 years of service
Retirement Age	Percentage of final compensation (%)						
50	5.5	11	16.5	22	27.5	33	
51	6.4	12.8	19.2	25.6	32	38.4	
52	7.3	14.6	21.9	29.2	36.5	43.8	51.1
53	8.2	16.4	24.6	32.8	41	49.2	57.4
54	9.1	18.2	27.3	36.4	45.5	54.6	63.7
55	10	20	30	40	50	60	70
56	10.32	20.64	30.96	41.28	51.6	61.92	72.24
57	10.63	21.26	31.89	42.52	53.15	63.78	74.41
58	10.94	21.88	32.82	43.76	54.7	65.64	76.58
59	11.25	22.5	33.75	45	56.25	67.5	78.75
60	11.57	23.14	34.71	46.28	57.85	69.42	80.99
61	11.88	23.76	35.64	47.52	59.4	71.28	83.16
62	12.19	24.38	36.57	48.76	60.95	73.14	85.33
63+	12.5	25	37.5	50	62.5	75	87.5

3. Methodology

Two approaches can be used to solve the problem of optimal retirement age:

3.1 Approach I

We take the following steps for Approach 1:

1. Table 1 shows the data for retirement age as it varies with benefit factor. This data is taken from CALPERS (2002).
2. Establish a mathematical relation between final compensation and retirement age from 50 to 63+ for 5, 10, 15, 20, 25, 30 and 35 years of service.
3. Plot behavior curve between final compensation and retirement age from 50 to 63+ for 5, 10, 15, 20, 25, 30, and 35 years of service.

   Based on CALPERS data, before age 63, the percent of final compensation depends both on retirement age and number of years of service. For age 63 and beyond, the benefit factor is fixed at 2.5% and the final compensation is only determined by number of years of service and not by retirement age any more.
4. The behavior curve plotted in step 2 is superimposed on the longevity curve (Chung, 2011) which is essentially a behavior curve between employees' retirement age and age of death.
5. A functional relation is then developed between retirement age and age of death.
6. Superimposing the two behavior curves (developed in Step 2 and Step 3) and using the mathematical equations developed in Step 1 and Step 4, we derive local optimal retirement ages for service of 5, 10, 15, 20, 25, 30 and 35 years respectively.
7. We plot number of years of service against the local retirement ages. From this, we then obtain the global optimal retirement age.

We use the benefit factors in Table 1 to calculate percentage of gross salary of an employee which is defined as the product of benefit factors and number of years of service. It is to be noted that the benefit factors vary with retirement age. Fig. 1 plots the relation between benefit factor and retirement age, which is show to be almost linear. The equation for this relation is:

Benefit Factor and Retirement Age: $y = 0.1026x - 3.8277$ (1)

The coefficient of determination (R^2) is 0.9325 and the correlation coefficient (r) is 0.9656.

To check the statistical adequacy of this equation, two parameters are calculated in general. One is r (Correlation coefficient) and the other is $s_{y/x}$ which is standard error of estimate. For a good fit, $r >= 0.8$ and $s_{y/x} <= s_y$ in which $s_{y/x}$ is the standard error of estimate and s_y is the standard deviation of the dependent variable y. These expressions are taken from literature (Ang & Tang, 2007). To get an optimal age of retirement this curve is superimposed with the longevity curve. The longevity curve is shown in Fig. 2 based on the data available in literature (Chung, 2011).

Figure 2: Longevity Curve (Chung, 2011)

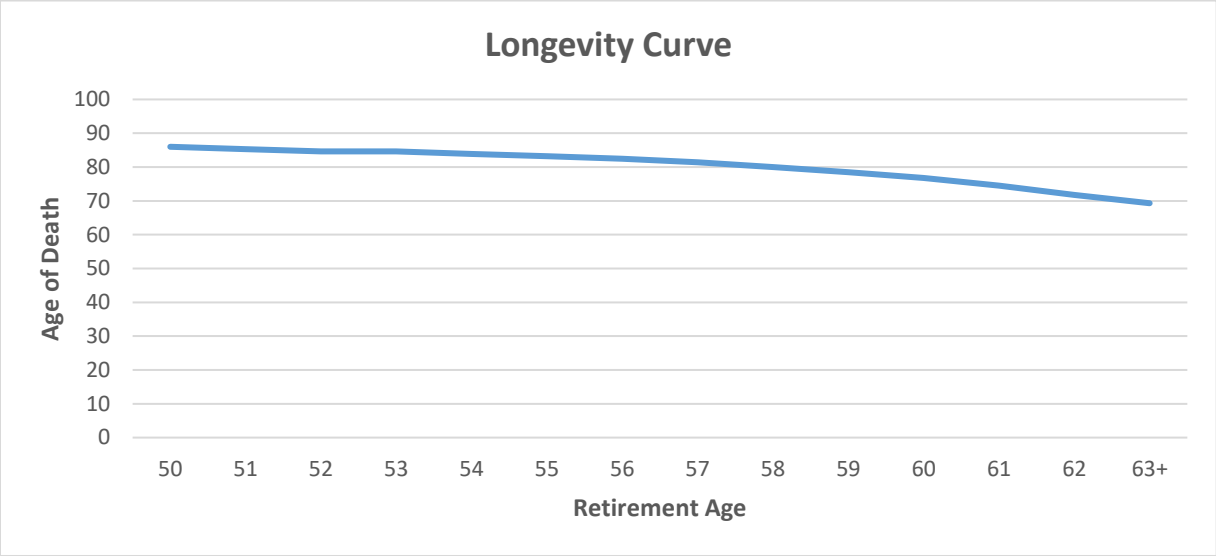
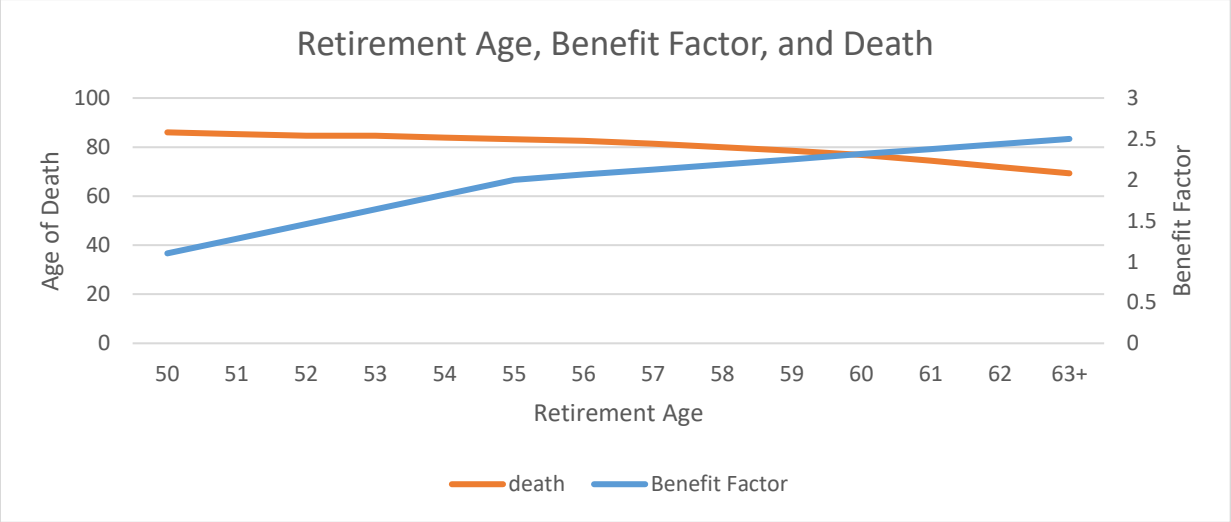


Figure 3 shows combined curve of benefit factor and longevity curve.

Figure 3: Retirement Age, Benefit Factor, and Death

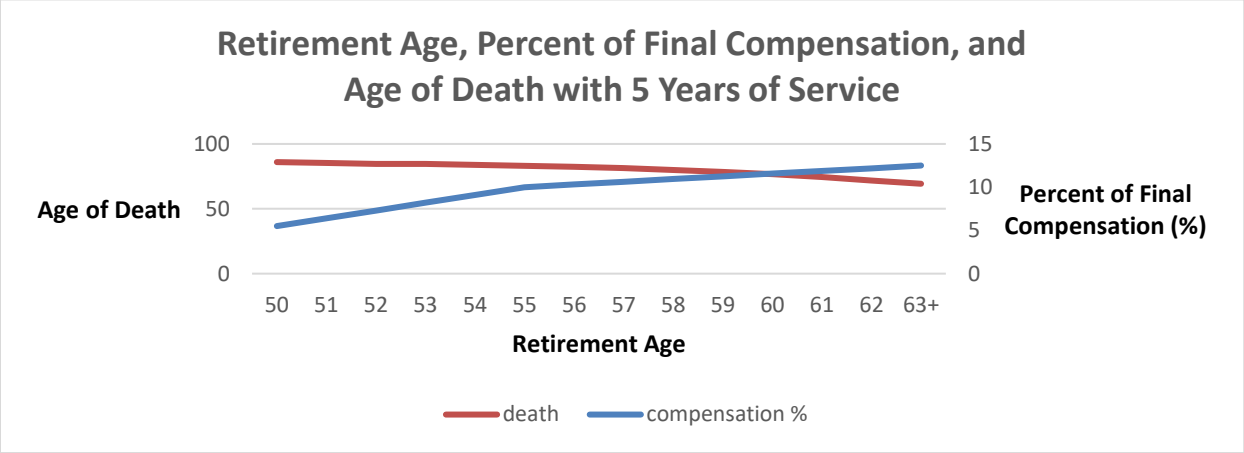


Using the benefit factor information, we plot percentage of gross salary against retirement age for service years of 5, 10, 15, 20, 25, 30, and 35 respectively. These are shown in Figs. 4-10.

A function relation has been developed for each of these plots based on the concept of regression analysis. For a good fit, the correlation coefficient r >= 0.8. The steps outlined in Approach I are implemented and the resulting Figures and Equations are shown below.

5 years of service:

Figure 4: Retirement Age, Percent of Final Compensation, and Age of Death with 5 year of Service

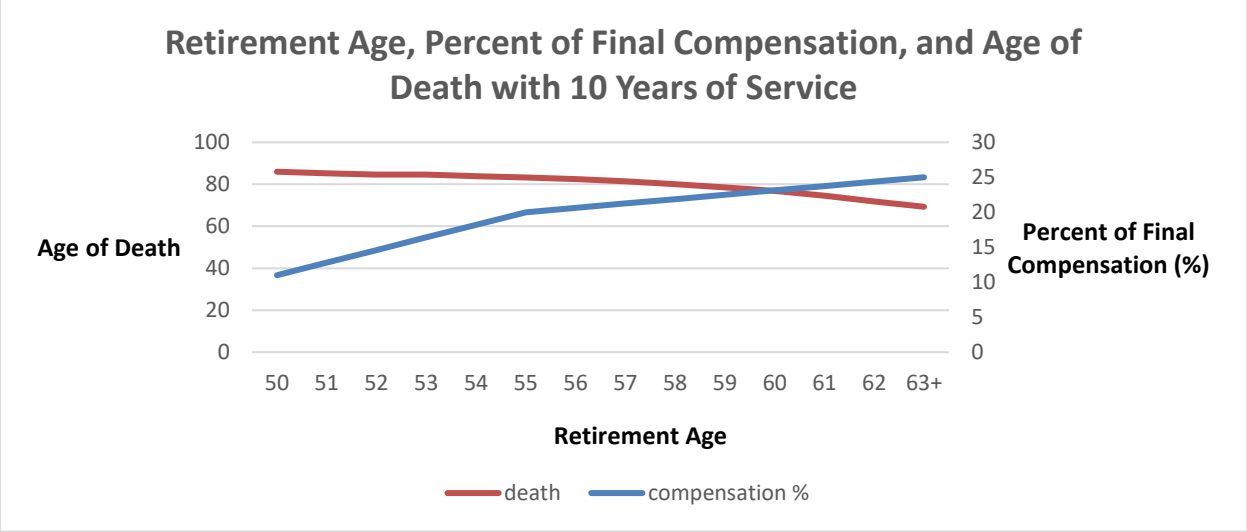


Compensation and retirement age: y=-20.54+0.54x (2)

R^2 = 0.9342

10 years of service:

Figure 5: Retirement Age, Percent of Final Compensation, and Age of Death with 10 year of Service

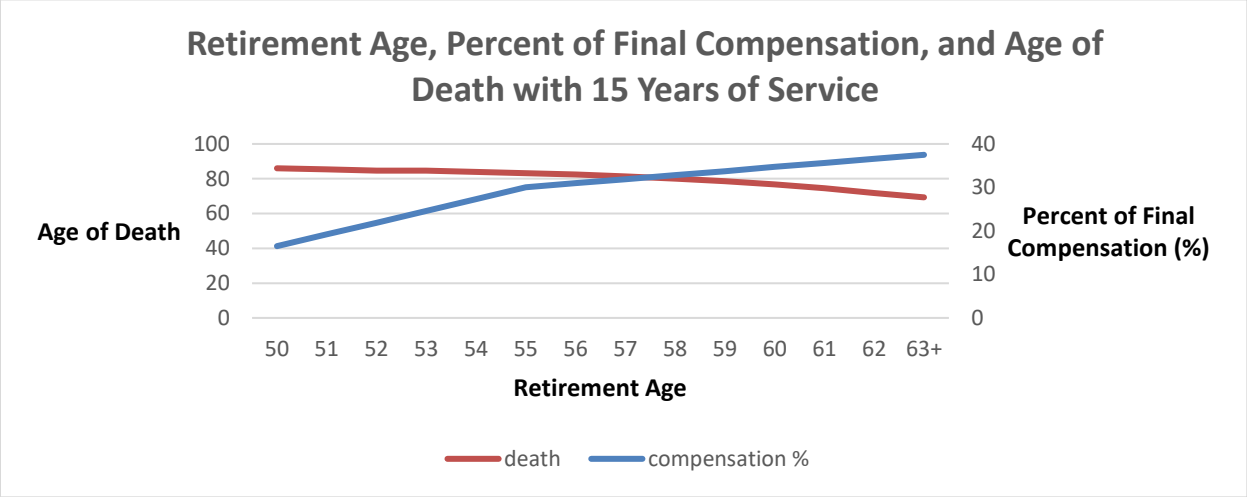


Compensation and retirement age: y=-41.08+1.08x (3)

R^2 = 0.9342

15 years of service:

Figure 6: Retirement Age, Percent of Final Compensation, and Age of Death with 15 year of Service

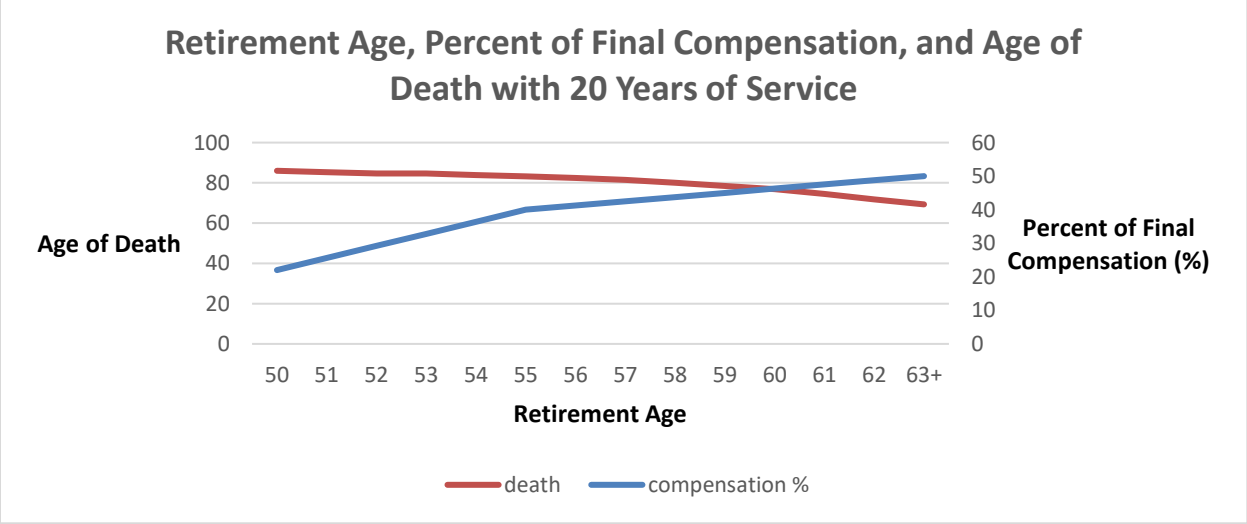


Compensation and retirement age: y=-61.62+1.62x (4)

R^2 = 0.9342

20 years of service:

Figure 7: Retirement Age, Percent of Final Compensation, and Age of Death with 20 year of Service

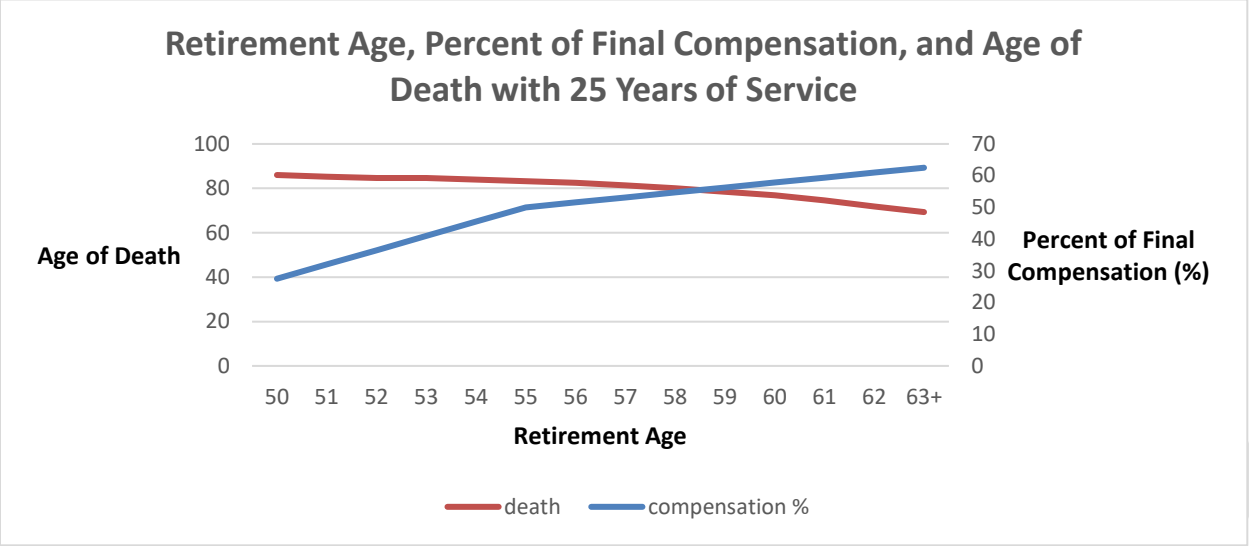


Compensation and retirement age: y=-82.17+2.16x (5)

R^2 = 0.9343

25 years of service:

Figure 8: Retirement Age, Percent of Final Compensation, and Age of Death with 25 year of Service

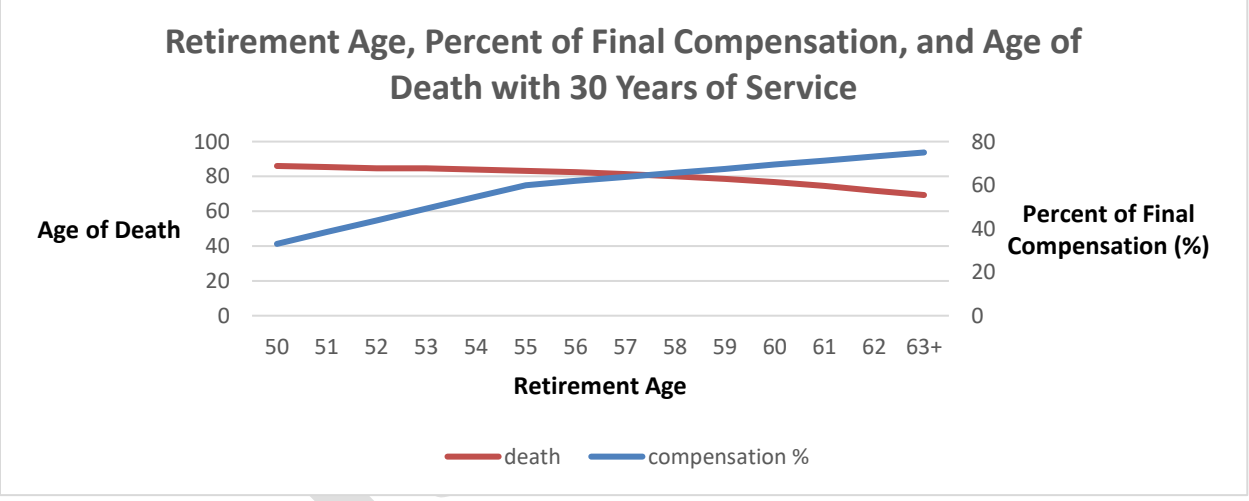


Compensation and retirement age: y=-102.71+2.69x (6)

R^2 = 0.9343

30 years of service:

Figure 9: Retirement Age, Percent of Final Compensation, and Age of Death with 30 year of Service

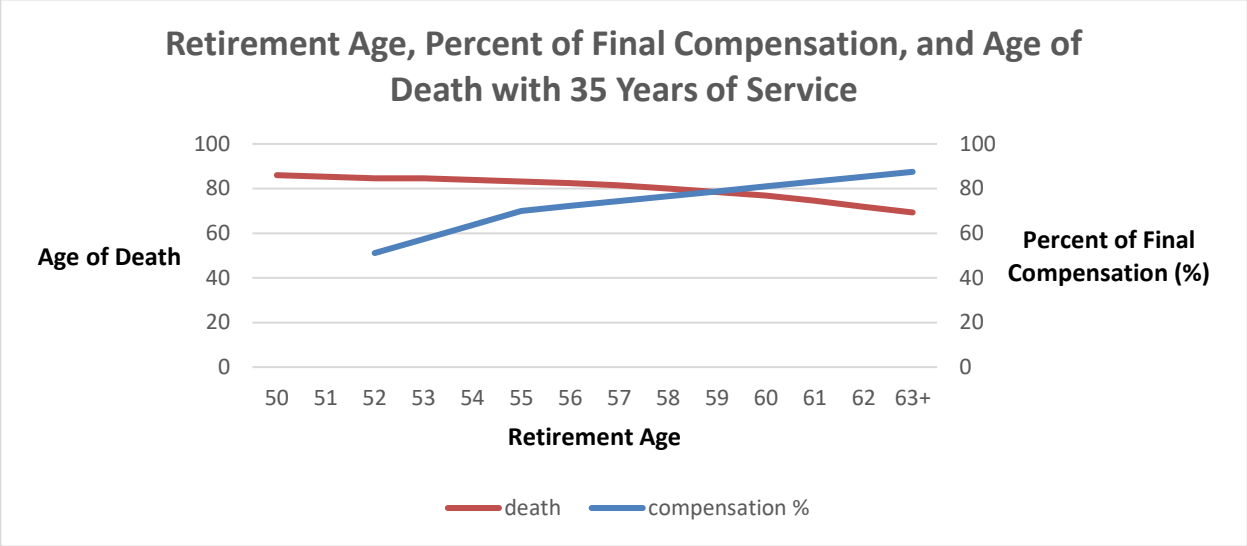


Compensation and retirement age: y=-123.23+3.23x (7)

R^2 = 0.9342

35 years of service:

Figure 10: Retirement Age, Percent of Final Compensation, and Age of Death with 35 year of Service



Compensation and retirement age: y=-108.13+3.16x (8)

R^2 = 0.9342

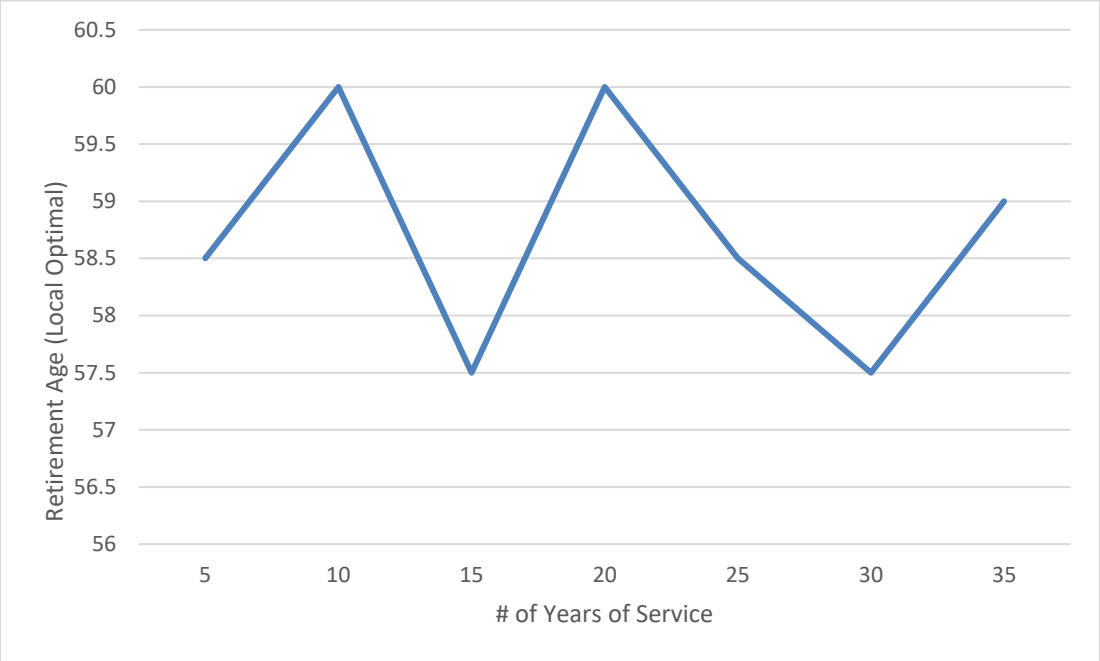
Figures 4 through 10 show that the optimal retirement age varies based on number of years of service. We calculate and tabulate those local optimal retirement age as below in Table 3.

Table 3: Number of Years of Service and Retirement Age (Local Optimal)

# of Years of Service	Retirement Age (Local Optimal)
5	58.5
10	60.0
15	57.5
20	60.0
25	58.5
30	57.5
35	59.0

We then plot the data in Table 3 in Figure 11 below.

Figure 11: Number of Years of Service and Retirement Age (Local Optimal)



We can conclude from Figure 11 that the optimal age of retirement is 60.

3.2 Approach II

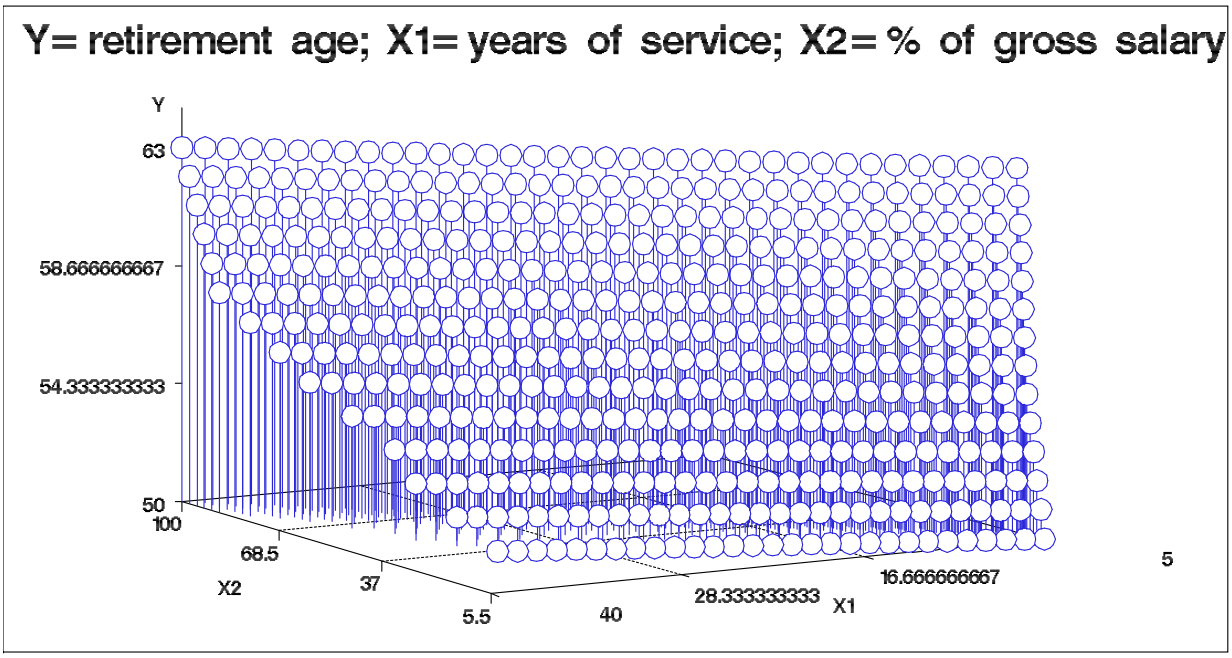
It is noted that Approach I treats data for 5 years of service and 10 years of service separately. In Approach II, both the percentage of gross salary and number of years of service are treated as independent variables (as x_1 and x_2) and the retirement age as a dependent variable (y). We first derive a relation between y, x_1 and x_2 by using the concept of multiple linear regression analysis. The following equation has been developed between y (retirement age), x_1 (years of service) and x_2 (% gross salary).

$$\mathbf{y = 56.8864 - 0.7723\, x_1 + 0.3795\, x_2} \qquad \mathbf{(9)}$$

The correlation coefficient for this regression equation is 0.489954.

We then use the principles of calculus to get an optimal value of percentage of gross salary x_1, optimal number of years of service x_2, and the corresponding optimal retirement age. The plot between the variables y, x_1 and x_2 is shown in Fig. 12.

Fig. 12. Variation of retirement age, years of service, and % gross salary



3.3 Discussion of Results

Table 4 presents the correlation coefficients (r) and the standard error of estimates ($s_{y/x}$).

Table 4: Correlation coefficients and standard error of estimates for regression equations developed

Eq. #	r	$S_{y/x}$	S_y
1	0.965636	0.007970438	0.120218933
2	0.965578	0.039852188	0.601094667
3	0.965565	0.079704377	1.202189334
4	0.965513	0.119556565	1.803284453
5	0.965488	0.159408753	2.404378667
6	0.965447	0.199260942	3.005473334
7	0.965422	0.239113134	3.606568001
8	0.965396	0.243456333	3.753223466
9	0.489954	$S_{Y/X1}$=0.214242 $S_{Y/X2}$=0.190994	0.128195765

Table 4 shows that the correlation coefficient for all the regression Equations (Eq. 1-8) is high. In addition, the condition that the standard error of estimate ($s_{y/x}$) is less than s_y is also satisfied. Hence, we can safely state that all the equations adequately represent the data for which these equations are developed.

From Fig. 11 we can see that the optimal retirement age based on Approach I is 60. Similarly, from Fig. 12 that uses multiple linear Regression analysis and Equation 9, we can see that the optimal retirement age is 56.8864 based on Approach II. The results are very close for both approaches.

The slight difference between the two sets of results from Approach I and Approach II is probably result from the fact that the multiple linear regression obtained in Equation 9 doesn't satisfy the adequacy tests for regression analysis. In Approach I the fitted relation for linear regression (Eq. 1-8) used between the dependent variable y and the independent variable x satisfies the adequacy tests, which show very high correlation coefficient r and $s_{y/x}$ is less than s_y. However, in Approach II the multiple linear regression (Equation 9) has a very low correlation coefficient r and $s_{y/x}$ is greater than s_y. It is expected that the results would be closer if a better fit for multiple linear regression is used.

4. Conclusions

Hence, our study shows that the optimal age of retirement is 60 based on CSU retirement system. It is to be noted that a mathematical approach developed to obtain retirement age based on certain given specifications (CALPERS data in this case). But our approach is very general and can be used for any practical system. Therefore, our research has lot of practical applications and is useful to both academia and general public.

Additional mandatory information

A. Compliance with Ethical Standards

There is no potential conflicts of interest related to my paper.

This research does not involve any human/animal participants.

We give consent to this Journal to publish my paper.

B. Funding

No funding was received for this research work.

C. Conflict of Interest

There is no conflict of interest for any author.

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REFERENCES:

1. Ang, A.H-S and Tang, W. (2007). Probability Concepts in Engineering. John Wiliey & Sons.
2. Chung, E (2011). Actuarial Study of life span vs. retirement age.
3. http://faculty.kfupm.edu.sa/coe/gutu...sc/Retire1.htm
4. CALPERS (1981). 2% at 55 Formula. Published by CALPERS.
5. Putcha, C.S. and Sloboda, B. (2015). A New Approach for a Forecasting Model in the Estimation of Social Security Benefits", (with B. Sloboda). 35th International Symposium on Forecasting, Riverside, CA, June 21-24.
6. Putcha, C.S. and Sloboda, B (2013). Predicting Unemployment Rate in The United States Since 2001. Chandrasekhar Putcha, Brian Sloboda and Kalamogo Coulibaly (Editors), Edwin Mellen Press.

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Appication Of Rapid Prototyping In Medical Field

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Abstract— Rapid prototyping (RP) is one of the fastest developing manufacturing technologies in the world today.Rapid Prototyping Technology can be used to convert 3D image into actual physical 3D Model.It is a quite different technology of manufacturing other than Lathe,Milling and other ManufacturingTechnology.The models which one cant prepare by conventional method that can be prepare by Rapid Prototping.In the medical field there is great use of this technology. This technique can be used to prepare medical tools and instrumentation.

Keywords— Rapid prototyping, STL File, MRI Scan,3D Image,SLA,SLS,FDM

1.INTRODUCTION

To compete in today's industry environment, companies must keep up with the leading technologies and processes and also push the boundaries and develop new and improved products and processes. Shortening the lead-time for introducing a new product to the market has always been important to maximize profits and competitiveness. Recent developments in Computer Aided Design (CAD) technologies have significantly reduced the overall design cycle. However, the manufacturing process of the production mold still relies on slow and expensive machining processes. The Manufacturing Industry is an area where time, efficiency and accuracy are the major driving forces behind innovation and research. The most competitive companies are those who continually reduce process times, increase efficiency and improve accuracy. Rapid Prototyping is an area that has and is continuing to reduce production time and increase efficiency and accuracy in developing and manufacturing prototypes compared to traditional prototype manufacture. The research development of Rapid Prototyping (RP) is to give the Rapid manufacturing the needed confidence to go on to customized/tailor made product. Investment casting is a combination of science, experience and art. Prior to final design and pattern construction, it is important to select an investment casting foundry and initiate communications. Typically, each foundry will have unique capabilities, processes and requirements. In addition, pattern specifications will vary with the selection of the metal alloy and the geometry of the part. If producing patterns for the foundry, it is critical that the foundry reviews the design so that it can recommend necessary design modifications to produce the highest quality part. The foundry can also make recommendations that reduce cost, time and weight, while improving cast ability and product performance. Additionally, FDM research is ongoing, so new process guidelines may evolve. The goal of this research is to formulate a generalized Mathematical Model for Optimum temperature & time with given multiple choices of various Shell thickness & RP Part volume and find the optimal Model equation in project for the manufactured any complicated shape regular & non regular in confidence level by using Design of experiment technique.
All the RP techniques employ the same basic five step process. The steps are as follows:
i. Create a CAD model of the design.
ii. Convert the CAD model in to STL format.
iii. Slice the STL model in to thin cross sectional layers.
iv. Construct the model one layer atop another.
v. Clean and finish the model.

The use of Rapid prototyping for medical applications although still in early days has made impressive strides. Its use in orthopaedic surgery, maxilla-facial and dental reconstruction, preparation of scaffold for tissue engineering and as educational tool in fields as diverse as obstetrics and gynaecology and forensic medicine to plastic surgery has now gained wide acceptance and is likely to have far reaching impact on how complicated cases are treated and various conditions taught in medical schools.

2. RP MEDICAL MODEL PRODUCTION

The procedure for making 3D medical models using RP technologies implies few steps:
• 3D digital image;
• Data transfer, processing and segmentation;
• Evaluation of design;
• RP medical model production;
• RP medical model validation.

2.1. 3D Digital Image

3D digital image can be obtained by using computer tomography - CT scanner or MRI data (see Fig. 1). These imaging technologies are used for modeling internal structures of human's body. Medical models made from this data must be very accurate and because of this they require a spiral scanning technique which allows to do full volume scanning. This makes possible to generate a high number of slices (recommended thickness 1-2 mm) and what is very important, the pixel dimension in each slice could be reduced depending on each case. Most CT and MRI units have the ability of exporting data in common medical file format - DICOM – digital imaging and communication in medicine.
Fig. 1 MRI unit

FIG 1:MRI UNIT

2.2. Data Transfer, Processing and Segmentation

After saving CT or MRI image data, they should be transferred to RP or RE laboratory.The scanned file should be informat of stl ot DIACOM. The next step is processing these data, which is a very complex and important step, After the data is exported in DIACOM file format, it needs to be converted into a file format which can be processed for computing and manufacturing process. In most cases the desired file format for Rapid manufacturing is .STL or sterolithographic file format. The conversion requires specialised softwares like MIMICS, 3D Doctors, AMIRA. These softwares process the data by segmentation using threshold technique which takes into the account the tissue density. This ensures that at the end of the segmentation process, there are pixels with value equal to or higher than the threshold value. A good model production requires a good segmentation with good resolution and small pixels.

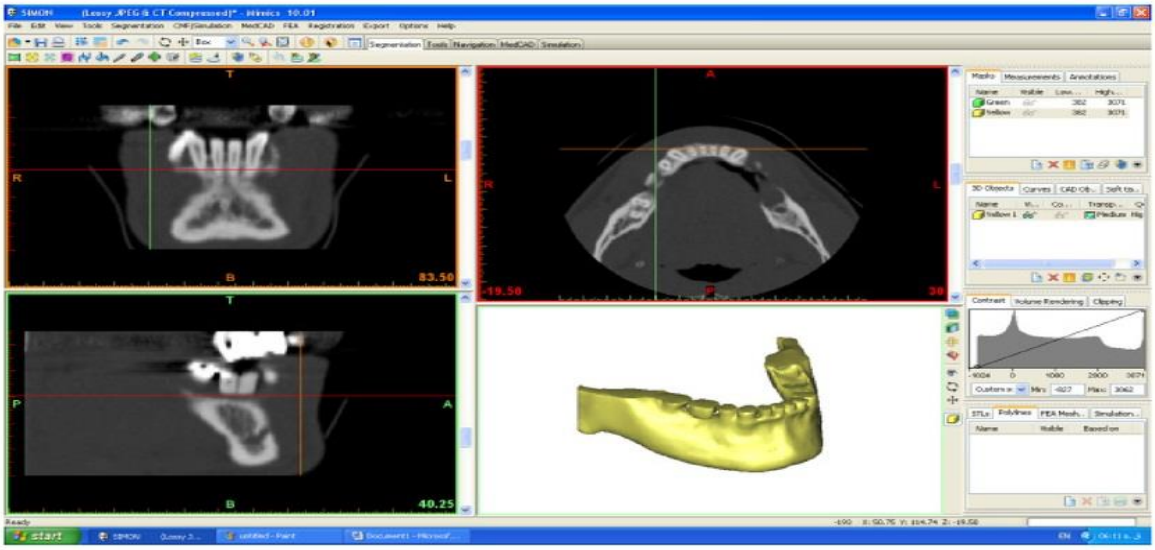

FIG 2:Software Package for Image Segmentation

2.3 Evaluation of Design

This step requires combined effort of surgeon, bio engineer and in some cases radiologist. It is important that unnecessary data is discarded and the data that is useful is retained. This decreases the time required for creating the model and also the material required and hence cost of production. Sometimes this data can be sent directly to machine for the production of model especially when the purpose of model is to teach students. The real use however is in surgical planning in which it is critical that the surgeon and designer brain storm to create the final prototype. There may be a need to incorporate other objects such as fixation devices, prosthesis and implants. The step may involve a surgical simulation carried out by the surgeon and creation of templates or jigs. This may require in addition to the existing converting softwares, computer aided designing softwares like Pro- Engineer, Auto CAD or Turbo CAD.

2.4 Additive manufacturing and production of the model

This step implies choosing the right RP technology according to the purpose of model itself as well as demanding accuracy, surface finish, visual appearance of internal structures, number of desired colors in the model, strength, material, mechanical properties,etc. Finally 3D virtual model in STL format should be inputted into the RP commercial software for production of 3D physical model (see Fig. 3). The quality of physical model is influenced, in the first place, by quality of input STL file but also by orientation of the model in RP machine and by choosing the right parameters for building the model in the same machine.

FIG3:RAPID PROTOTYPING MACHINE

3. Rapid prototyping applications

- Improved quality – existing medical products, devices, parts, equipment, etc. can be improved in quality with possible objectives being stronger, lighter versions etc.
- Patient specific models – development of patient specific parts, e.g. prosthetics, dental products, etc.
- Reduced time to market – is extensively used to reduce the time taken for products to reach the market
- Training aids – prototypes/models can be developed to practice complex operations on models first, rather than "test" on live patients (realistic effects such as blood type substances for specific cuts can be added)
- aid the training of new medical staff
- for educating patients as to treatment they will be undertaking
- User trials – conduct user trials through offering different versions of medical products, which are quickly 3D printed and see which ones perform best
- Design and development of medical devices and instrumentation:-This is the field where applications of RP show the best results. It specially applies to hearing aids but also to other surgical aid tools.
- **Great improvements to the fields of prosthetics and implantation**. RP techniques are very useful in making prostheses and implants for years. The ability to quickly fit prosthesis to a patient's unique proportions is a great advantage. The techniques are also used for making hip sockets, knee joints and spinal implants for quite some time. Both the release of and the improvement of the properties of used materials have had a significant influence on the quality of prostheses and implants made by RP. One interesting example is maxillofacial prostheses of an ear which is obtained by creating a wax cast by laser sintering of a plaster cast of existing ear. Due to RP technologies it is very easy to manufacture custom implants. The made model could be used as a negative or a master model of the custom implant. Many researchers explored new applications of RP in this field
- **Planning and explaining complex surgical operations.** This is very important roleof RP technologies in medicine which enable presurgery planning. The use of 3D medical models helps the surgeon to plan and perform complex surgical procedures and simulations and gives him an opportunity to study the bony structures of the patient before the surgery, to increase surgical precision, to reduce time of proceduresand risk during surgery as well as costs (thus making surgery more efficient). The possibility to mark different structures in different colours (due to segmentation technique) in a 3D physical model can be very useful for surgery planning and better understanding of the problem as well as for teaching purpose. This is especially important in cancer surgery where tumor tissue can be clearly distinguished from healthy tissue by different colour. Surgical planning is most often done with stereolithography (SLA) where the made model has high accuracy, transparency but limited number of colours and 3DP (for more colored models, presentation of FEA results).
- **Design and manufacturing biocompatible and bioactive implants and tissueengineering.** RP technologies gave significant contribution in the field of tissue engineeringthrough the use of biomaterials including the direct manufacture of bioactive implants. Tissue engineering is a combination of living cells and a support structure called scaffolds. RP systems like fused deposition modelling (FDM), 3D printing (3-DP) and selective laser sintering (SLS) have been proved to be convenient for making porous structures for use in tissue engineering. In this field it is essential to be able to fabricate three-dimensional scaffolds of various geometric shapes, in order to repair defects caused by accidents, surgery, or birth. FDM, SLS and 3DP can be used to fabricate a functional scaffold directly but RP systems can also be used for manufacturing a sacrificial mould to fabricate tissue-engineering scaffolds.

CONCLUSION

Technologies are definitely widely spread in different fields of medicine and show a great potential in medical applications. Various uses of rapid prototyping within surgical planning, simulation, training, production of models of hard tissue, prosthesis and implants, biomechanics, tissue engineering and many other cases open up a new chapter in medicine. Due to rapid prototyping technologies doctors and especially surgeons are privileged to do some things which previous generations could only have imagined. However this is just a little step ahead. There are many unsolved medical problems and many expectations from rapid prototyping in this field. Development in speed, cost, accuracy, materials (especially biomaterials) and tight collaboration between radiologists, surgeons and engineers is necessary and so are constant improvements from rapid prototyping vendors. This will help rapid prototyping technologies to give their maximum in such an important field like medicine. Benefits resulting from the use of rapid prototyping equipment may include:

- Reduction of current operating costs (including labor, quality, purchasing, etc.)
- Improved sales and marketing due to the ability to respond to customer requests for bids with actual models
- Improved product development, including improved customer satisfaction and improved product manufacturability
- Improved process development as lead times and costs are reduced, leading to more iterations

REFERENCES:

[1] Schenectady Materials and Processes Laboratory, Inc & Biomet Inc.
[2] Howmet Research Corporation, communication to Stratasys, INC.-June 1998. ABS-P400 MSDS Sheet, November 4, 1994. EPA Handbook, Control Technologies for Hazardous Air Pollutants, June 1991.
[3] Dilip Sahebrao Ingole; Abhay Madhusudan Kuthe,‖Rapid Prototyping –a Technology transfer approach for development of rapid tooling‖; Rapid Prototyping Journal Vol.:15, No.:4, July 31,(2009),Pg 280-290.
[4] Loose, K., Investigation of Optimum Process Parameters for Burnout of SLA Parts, Institute of Industrial Science, University of Tokyo.
[5] Alain Bernard, Jean-Charles Delplace, Nicolas Perry and Serge Gabriel; ‒Integration of CAD and rapid manufacturing for sand casting optimization‖; Rapid Prototyping Journal; Vol9, No.5, (2003), pp. 327–333
[6] C. W. Lee, C. K. Chua, C. M. Cheah, L. H. Tan, C. Feng; ‒Rapid investment Casting: direct and indirect approaches via fused deposition modeling‖; International Journal Adv Manufacturing Technology, (2004), vol23: pg 93–101.
[7] S. Jones, C. Yuan; ―Advances in shell molding for investment casting‖; Journal of Materials Processing Technology; vol.135, (2003), pg 258–265.
[8] W.L. Yao and Ming C. Leu; ―Analysis of shell cracking in investment casting with laser stereolithography patterns‖; Rapid Prototyping Journal; Volume 5, Number 1,(1999),pg 2–20. 109
[9] D.K. Pal, Dr.B. Ravi; ‒Rapid tooling route selection and evaluation for sand and Investment casting‖; Virtual and Physical Prototyping Journal; 2(4), pg197-207, (2007).
[10] Rapid Prototyping (principles & applications)-Rafiq Noorani,Los Angeles, CA, Jhon wiley & sons, INC.
[11] Rapid Prototyping & Manufacturing- Paul F. Jacobs, Mc graw-hill publishers.
[12] Rapid Investment Casting via FDM Journal of advanced Manufacturing Technology C.lee, C.K.Chua Vol23,(2005),Pg 29-101.
[13] Che Chung Wang, Shr-Shiung Hu; ‒Optimizing the rapid prototyping process by Integrating the Taguchi method with the Gray relational analysis‖; Rapid Prototyping Journal; Vol 13,No.5, (2007), 304–315.
[14] D. C. Montgomery, Design and Analysis of Experiments (second ed.), Wiley, New York, 1990.
[15] Harish Kumar Banga,Parveen Kalra,R.M.Belokar,Rajesh Kumar,'Rapid Prototyping Application in Medical Sciences',IJETCAS,pg-416-420
[16] Vaibhav Bagaria, Darshana Rasalkar, Shalini Jain Bagaria and Jami Ilyas,' Medical Applications of Rapid Prototyping - A New Horizon'www.intechopen.com.
[17] Robert L. Mason, Richard F. Gunst, Dallas, Texas, .James L. Hess. Statistical Design and Analysis of Experiments with Applications to Engineering and Science (Second Edition), A John Wiley & son's publication, 2003

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Importance of Yoga and Spirituality in Modern Lifestyle

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Abstract- Yoga is emerging as an integral need to keep oneself fit in today's busy lifestyle. Yoga gives them peace of mind that's why people are devoting some time from their busy daily schedule to practice yoga. Maharishi Patanjali's Ashtanga Yoga inspires mind control for spiritual growth along with physical elevation. It is not only for sages, sanyasis living on the Himalayas but also guidance for all personnel leading a family life.

There are many types and benefits of practicing yoga according to Patanjali's Yogasutra.

Yoga has eight major parts:

- Yama
- Niyama
- Asana
- Pranayama
- Pratyaharaya
- Dharna
- Dhyana
- Samadhi

It's a proven fact that by various yoga postures, special breathing exercises and meditation in Yoga, stresses and tensions are relieved. Yoga relaxes the mind by removing various distractions from mind and stabilizes the brain in focusing on a particular job.

We humans are attracted to something only when we benefitting from it. The way we are attracted to yoga is an indication that yoga has many benefits. Yoga not only gives strength and satisfaction to our body but also enhances our brain and spiritual power. Yoga is beneficial for everyone be it a man, woman, child, youth or elderly.

Key Words : Lifestyle, Yogasutra, Physical and Spiritual Advancement, Mental Control, Spiritual Force, Inner Peace

Introduction:

Everyone is finding himself busy in this extremely materialistic world and modern lifestyle. Due to his/her busy schedule he/she has lost his/her comfort, peace of mind and happiness. He does not have time to pay attention even to his health. In spite of pressure, stress, depression, chaos, sickness, insomnia, despair, failure, work, anger, greed, fascination, arrogance, jealousy etc., nowadays he is bound to lead a difficult and a painful life. Today, he has become a victim of all kinds of evils along with adulteration of food, water, air and sound pollution. As a result, today he is surrounded by many physical diseases as well as mental imbalances, worries, sadness, listlessness and malicious thoughts. His peace of is mind is disturbed.

Our Indian mythological yoga system can help to face these situations with firmness. If we look carefully, the main objective of human existence is only yoga. Making Yoga practice an integral part of life will assure the recovery of lost human power and will teach the art of living life with eternal truth again. Yoga is a complete knowledge of life itself. Yoga liberates us from all our physical, mental and emotional stresses and diseases. It is committed to fullness and unending bliss.

Spiritual Outlook

In 2nd chapter and 5th Verse of Srimad Bhagwat Gita, Lord Shri Krishna defines Yoga as ----"Karmasu Kaushalam" means "The Name of Skill of Actions".

Explaining it in Maharishi Patanjalikrit Yoga philosophy, it is said ----

"Yogaschita Vritti Nirodha:

Control of mind is the sum. In this, the yoga practitioner establishes himself in his proper form i.e. Yoga is another name for discipline.

Ashtanga Yoga of Maharishi Patanjali inspires mind control for spiritual growth along with physical elevation. It is not only a path for saints and sages living on the Himalayas, but also for each and every person.

Yoga is life and carnal pleasure is evil.

In Yoga, Shiva is considered as the first Yogi or Adi Yogi and the first Guru or Adi Guru. After Lord Shiva, Yoga is considered to have emerged from Vedic sages and saints. Later lord Krishna, Lord Mahavir and Lord Buddha took it and expanded Yoga in their own way.

Finally Maharish Patanjali gave it a systematic shape and later on followers of Siddhpant, Shaivism, Nagpant, Vaishnavpant and Shaktapanth expanded yoga in their own way.

As per Mandal-1, Sukta-18, Mantra-7 of Rigasamnhita:

"यस्मास्ते न सिध्यति यज्ञों विपश्चितशचं I स धीनां योगमिन्वति"

No sacred rituals of scholars are complete without Yoga.

What is yoga?

Yoga is the restraint of mind and is fully immersed in duty.

According to **Rigasamhita's Mandal-1, Sukta-5, Mantra-3**

"स घा नो योग आभुवत स राये स पुरं ध्याम। गमद वाजेभिरा स नः "

It means that God may be oriented towards our Samadhi but we will get the benefit of Samadhi, Vivekhyati, Ritambhara and Pragya by his mercy only, so O divine god please come to us with the perfections of the divine achievements.

Main parts of yoga:

The eight major parts of yoga are:

Yama, Niyam, Asana, Pranayama, Pratyahara, Dharna, Dhyana and Samadhi.

- **Yama:**

  There are five Yams - Ahimsa, Satya, Astya, Brahmacharya and Aparigraha.

- **Niyam:**

  There are five rules - Defecation, Satisfaction, Lie, Celibacy and Imperfection

- **Asana**:

  Asana is the name of sitting easily & happily without any movement. According to his ability as practitioner can sit in position in which he/she feel comfortable and can sit for a long time without any agony or agitation in a steady manner and this comfortable sitting position is a suitable **Asana** for him.

  While sitting, all three body parts, the head, throat and spinal cord should be straight and stable. When the Asana is perfected, there is no affect of heat, cold etc conflicting elements on the body, the body has the power to bear them all without any pain. The seeker's concentration can't be disturbed in any way if he has perfected the Asana.

  The Asana is the physical posture in which the stability of the body increases and the mind becomes calm steady & peaceful. Purification of the nerves, improvement of health, enhancement of bodily spirit and spirituality is achieved by perfecting the asana.

  Our body which is made of 5 elements (Aakash, Vaayu, Jal Agni and Prithwi) always remains healthy by practicing Asanas. The inner power awakens; all chakras open & mind is able to concentrate properly and as a result his/her working ability increases manifolds.

  The practice of Asanas plays an important role in all-round development of life and success in each and every task undertaken. Many times in life, a lot of problems arise due to adverse conditions. The practitioner has the power to face them easily.

Types of Yoga

In today's busy world, yoga is the only way that can keep us healthy. According to our physical ability, if we take half to one hour to serve our body in the morning at Brahma Vela, then we will benefit greatly.

The body is the best means of Sadhana. Someone has rightly said - **Jaan hai to jahan hai.**

Goswami Tulsidasji has also described in the **Ramcharitmanas** - the first happiness is a totally fit body. Regular practice of Yogasanas will lead to continuous improvement in health and the body will be infected with diseases. A lot of time should be devoted to Yogasanas.

Men, women and children of all ages can do it with pleasure. Only one firm resolution is required. Once a good habit of yoga is practiced, then all the work of the world will be left behind, but the practice of yoga will not be missed.

Some simple types of Yogasan are:

SL	Types Of Asanas	Benefits of Asana
1	**Padmasana**	Excess fat, body weight balanced, Divine meditation in wisdom posture, elasticity in thighs, laziness and constipation, Strengthen digestive power
2	**Yog Mudra**	Obesity is removed, freedom from all stomach disorders, spinal cord strengthened.
3	**Tulasana**	Bodily fat is removed, freedom from all stomach disorders, mildness in spinal cord, lightness in body, full body balance.
4	**Ardha Chandrasan**	Digestive system functioning smoothly, relieves stomach disorders, spinal flexibility, relieves waist pain
5	**Trikonasana**	Digestive system strengthened, liver and colon gland affected
6	**Surya Namshkara**	The 12 different positions have many benefits. Every external and internal organs of the body are positively affected, spinal cord strengthened, spleen elastic, digestive system and vascular system strong, relief from low blood pressure
7	**Shavasana**	A state of complete relaxation, relieving stress, calming the mind, experiencing bliss, relieving tiredness, relieving high blood pressure
8	**Tadasana**	Freedom from sluggishness, energetic body, helpful in raising the height of children, keeping the navel in its place, smooth transmission of blood, relieving waist and back pain, strengthening of spine, knees, ankles, buttocks, abdomen, shoulders, hands
9	**Naukasana**	Flexibility in spine, shoulders, back, extremities, refreshing, free from obesity
10	**Kamar Chakrasan**	Elasticity in the waist, freedom from obesity, relaxed and energetic extremities
11	**Jaanushirasana**	Blood excursions near the cervix, relieve leg pain, relieve joint pain
12	**Paschimottanasan**	Strength in hands and legs, spinal cord flexible, shoulders strong, digestive system strong, feeling of good appetite, free from obesity, elastic body
13	**Konnasana**	Beneficial in cervical creation, elasticity in the waist, arms, legs, shoulders strong, constipation release
14	**Goumukhasana**	Shoulder and knee strengthened, Strengthening of the brain, Mental balance, Strengthening the spinal cord

15	**Vajrasana**	The stomach should be empty. But after eating this asana, it also benefits a lot. Digestive system improved, Strengthened spinal cord, Relieve pain of waist and shoulders, Mental balance, Mind in God meditation
16	**Usthasana**	Stability in thought process, control in mental peace, strong chest muscles, flexibility in spinal cord, excess fat removal
17	**Supt Vajrasana**	No pain in knees, shoulders and waist, navel in its place, increased activity of kidneys, mental balance, free of obesity
18	**Sashankasana**	Nerves system strengthened, Relaxed, Mental peace, a, Renunciation of anger, jealousy and ego, Surrender to God.
19	**Sthilasana**	Relaxation, deep sleep, unrestrained mind release
20	**Sarpasana**	Removes obesity, strengthens digestive system, experiences good appetite, makes the body supple
21	**Bhujangasana**	Neck and spinal cord are strong and flexible, beneficial in the functions of kidneys, liver, uterus, stomach, lungs, heart and thyroid, relieve back pain, suppleness in body, beneficial in throat
22	**Shalabhasana**	Avoidance of obesity, high blood pressure and heart disease, strengthens spinal cord, increased lung function
23	**Dhanurasana**	Strengthens the kidneys, back and buttocks. Metabolism and muscle growth, strong and elastic, lungs, kidney, liver, intestines, spleen, and stomach.
24	**Halasana**	Nerves System strong, spinal cord elastic, thyroid gland benefited, obesity reduced, blood circulation smooth
25	**Uppasan**	Digestive system strong, obesity free hands and feet, Navel in place, Strengthens the nervous system

26	**Makarasana**	Strengthens the kidneys, liver and intestines, relieves indigestion and constipation, removes stiffness of the legs and back and increases the elasticity of the spinal cord, relaxes the back, is beneficial in strengthening the knee, buttocks and spinal discs. Remove all diseases, reduce obesity, high blood pressure and release from heart disease, pulse affected institutions
27	**Udar-Pawan Muktasan**	Heart and lungs are strong, relieve gas and acid, all stomach diseases go away, blood transmission, neck flexibility
28	**Sarwangasana**	Increase in activation of thyroid and pituitary glands, relieve tiredness, relieve sadness, beneficial in neck, eyes, ears, lungs, blood supply to neck and head, increase muscular function, spinal flexion

- **Pranayam :** Control of Inhalation and Expiration is Pranayam,

  It is said in Atharvaveda –

  "प्राणापानौ मृत्योर्मा पात स्वाहा"

  Means that both prana and apan protect me from death.

Yoga and Spirituality:

Ashtanga Yoga

The four parts of Ashtanga yoga are as follows:

- **Dharana** – It means holding the beliefs by which the desired condition can be attained. These are the conditions of happiness, health, peace, contentment, fulfillment, bliss which are addressed by spiritual possessions. Man is his own creator of fortune.

- **Pratyahar** - In the absence of distractions, senses and mind (Mann) almost becomes one and this state of a body is called Pratyahar. Isolating the senses from external distractions and merging them with the mind is Pratyahar. Pratyahar controls the senses.

- **Dhyana** - Attaining full concentration by detaching one's mind from all kinds of external and internal disturbance is called Dhyana. All tasks can be achieved with Dhyana. Life is meaningful only when we make it joyful, peaceful, musical and easy. All physical diseases and mental stress are eliminated by Dhyana. The sounds of sweetness and harmony are echoed by Jeevan Veena. All types of impulses and charges become peaceful and mind's restlessness ceases. It is through meditation that the seeker reaches the spiritual peak. It is absolutely necessary for a seeker to be honest, sincere and passionate.

- **Samadhi** – When in Dhyana, a person attains total control of his mind, body and soul and is totally oblivious of his surroundings then this Dhyana becomes Samadhi. For achieving grace of god and experiencing truth, getting enlightened and attaining bliss, we must thank the almighty god heartily.

Therefore, it is our duty and responsibility to keep the body and mind healthy by practicing yoga on a regular basis.

The main benefits of Yoga

It is a proven fact that stresses are relieved by yoga postures, meditation, yoga and special breathing exercises of yoga. Yoga provides stability by removing the mind from various distractions and helps it to concentrate on a particular work.

We are attracted to the thing only when we are benefitting from it. The way we are getting attracted towards yoga is an indication that yoga has many benefits. Yoga not only gives strength and satisfaction to our body but also to the mind and spiritual power. Women, men, children, youth or elderly, Yoga is for everyone and beneficial to everyone.

Changes can be made in yoga according to the body's capabilities and elasticity. In any case, yoga is beneficial.

Yoga of Mind and Emotions

It is very important to have positive thoughts in life. Pessimistic thoughts lead to failure. Yoga brings positive energy to the mind. Yoga gives spiritual strength and the mind gets rid of anxiety, contradiction and despair. By yoga the mind gets eternal peace and comfort which leads to happiness and enthusiasm. It has a direct impact on personality and health.

- **Stress relief**

  Stress is a disease in itself that invites many other diseases. Medical science also accepts this fact. An important advantage of yoga is that it provides relief from stress. In yoga posture, meditation and by special breathing exercises of yoga, stresses are relieved and are a proven fact. Yoga provides stability by removing the mind from various distractions and helps in stabilizing the mind in specific tasks. Being stress free has positive effects on body and mind. The ability to work also increases.

- **Development of mental abilities**

  Memory and intellectual ability are considered to be the main tools of a progressive life. Yoga develops mental abilities and also has a qualitative effect on memory. Yoga posture and meditation helps in directing and concentrating the mind towards the goal. The memory power increases with a concentrated mind. Questions based on logical abilities are asked in competitive examinations. Yoga also develops reasoning power and increases skill. Due to the qualitative effects of logical power and efficiency through the actions of yoga, confidence also increases.

- **Elasticity in Body**

  Yoga makes the body strong and flexible. Yoga keeps muscles fit and body balanced. Having a strong, balanced and elastic body also increases work efficiency. Some yoga postures also strengthen the bones of the body. It also reduces the chances of bone related disease.

Health and yoga

Yoga keeps the body physically fit and removes many types of physical and mental troubles. Yoga makes breathing easier. By taking deep breaths during yoga, the body relieves tension. Blood circulation becomes good & smooth with yoga and harmful toxins are extracted from the body. It gives relief from fatigue, headache, joint pain and also helps in maintaining normal blood pressure.

Conclusion

It's quite obvious that yoga has many benefits in daily life. With Yoga, a person can keep himself healthy even in this busy and hectic life. If a person adopts yoga in time, then he gets freedom from various diseases and his life and its quality will also increase.

REFERENCES:

1. A History of Yoga (1982) Rottledz ISBN 0-7100-9258-X Panna by Vivian Worthington. 29.
2. Dan Laesthaus. Buddhist Phenomenology: A Philosophical Investigation of Yogakara Buddhism and the Cheng Wei-Shih Lun. (Rautledz) Published 2002. ISBN 0-7007-1186-4.PW 533
3. Barbara Stoller Miller, Yoga: The Discipline to Freedom Yoga Sutra is attributed to Patanjali, with text translation, commentary, introduction and vocabulary keywords. University of California Press, 1996, page 8.
4. Hatha Yoga: This passage is written by Theori und Praktis Mikel Burley (p. 16).
5. Bhagavad-gītā, including an entire chapter (ch. 6) devoted to the practice of traditional yoga. This Gita introduces the famous three types of yoga, such as 'Jnana' (knowledge), 'action' (karma) and 'love' (devotion). "Flood, p. 96
6. Patañjali (2001-02-01). "Yoga Sutras of Patañjali" (etext). Studio 34 Yoga Healing Arts. Access date 2008-11-24.
7. Samuel, Geoffrey (2008). The Origins of Yoga and Tantra. Cambridge University Press. Page 4. ISBN 9780521695343.
8. Werner, Karel (1998). Yoga and Indian Philosophy. Motilal Banarsidass Publ. Page 103. ISBN 9788120816091.
9. Werner, Karel (1998). Yoga and Indian Philosophy. Motilal Banarsidass Publ. Page 103. ISBN 9788120816091.
10. Zimmer, Heinrich (1972). Myths and Symbols in Indian Art and Civilization. Princeton University Press, New Ed edition. Page 168. ISBN 978-0691017785.

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Brain Wave Solution to the Modified Schrodinger Equation

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Abstract: What is your perception of reality? Is it same as my perception of reality? What defines perception? Does perception like ice cream? And at last but not least, does perception even exist? Questions like these are the basis of the modern doctrine of consciousness. After a long time of research we have not been able to understand much about consciousness, however we humans have been able to find a fundamental property of consciousness without even fully understanding the 'consciousness' itself. The fundamental property of consciousness is the memory.
In Bohmian mechanics (Bohm's interpretation), the pilot wave acts as a navigator for a particle, and so happens in human brain. In this paper we contemplate the bohm's interpretation and its indirect relation with consciousness, by showing its relation with human memory. The bohm's interpretation is an interpretation of modified Schrodinger equation, which itself is connected with the memory of quantum system.

Keywords: Quantum Mechanics, Bohmian Mechanics, Pilot Wave, Consciousness, Thermal Memory, Physics & Brain, Nature of Reality, Schrodinger Equation, Modified Schrodinger Equation, de Broglie and Bohm Interpretation.

1. INTRODUCTION

The Bohm's interpretation, with a system of N particles is described by the wave function and the configuration q_k, or we can say by the actual position of the quantum objects. This implies that bohm has to add an "equation of navigation" or "equation of motion" for the positions to the formalism. Let's assume a wave function $\Psi=R\exp(iS/\hbar)$, the navigation equation for the position q of a spin-less particle in 1-particle case takes the form:

$$\frac{dq}{dt} = \frac{\vec{\nabla}S}{m} \tag{1}$$

This equation is dependent on the initial configuration to fix the motion uniquely. The generalization of >1 particle case including spin is straightforward[1]. Using 'Born rule' i.e. $\rho= |\psi|^2$ to give initial conditions for the position, the equation of continuity

$$\frac{d\rho}{dt} + \nabla j = 0 \tag{2}$$

Using the usual quantum mechanical probability current

$$j = \frac{\hbar}{2mi}[\Psi^*(\nabla\Psi) - (\nabla\Psi^*)\Psi \tag{3}$$

$$= \rho\frac{\nabla S}{m} \tag{4}$$

ensures that the position remains $|\Psi|^2$ distributed. We do this to ensure that in terms of position any measurement yields exactly the result of standard formalism, so that the navigator equation is consistent with requirements of quantum mechanics.
In ordinary Quantum mechanics the probability current refers to the probability to measure a certain position. In Bohmian mechanics it is viewed as the probability of the particle to be at a certain position; independent of any measurement.

The work on this doctrine was originally did by Louis de Broglie in 1920 [2] i.e. why it is also called "de Broglie Bohm" theory (interpretation). Which [1] presents a version called "Bohmian Mechanics", the books by Bohm and Hiley [3] are more closer to the orginal research of bohm from1952 [4]—also called ontological interpretation of Quantum mechanics.

2. Brain wave activity as solution to the Modified Schrodinger equation

To start, first we derive Modified Schrodinger Equation by contemplating the thermal phenomena. The thermal history of the system (universe) can be described by the generalized Fourier equation [5] [6]

$$q(t) = -\int_{-\infty}^{t} K(t-t')\nabla T(t')dt' \tag{5}$$

Where,

$K(t\text{-}t')$={*thermal history (thermal memory)*

$\nabla T(t')dt'$={*diffusion*

In Eq. (5) q(t) is the density of the energy flux, T is the temperature of the system.

$$K(t-t') = \frac{K}{\tau}\exp\left[-\frac{(t-t')}{\tau}\right], \tag{6}$$

Where τ denotes relaxation time,

$$K(t-t') = \begin{cases} K\delta(t-t') & (\text{diffusion}) \\ K = constant & (\text{wave}) \\ \frac{K}{\tau}\exp\left[-\frac{t-t')}{\tau}\right] & (\text{hyperbolic diffusion}) \end{cases}$$

The hyperbolic diffusion rewritten, becomes

$$\frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau}\frac{\partial T}{\partial t} = \frac{D_T}{\tau}\nabla^2 T \tag{7}$$

For τ→ 0, the Eq. (7) becomes the Fourier thermal equation

$$\frac{\partial D}{\partial t} = D_T\nabla^2 T \tag{8}$$

and D_T is thermal diffusion coefficient. The systems with very short τ have very short thermal history. For τ→∞ Eq. (7) has the form of *ballistic thermal equation.* The ballistic phonons or electrons are those for which τ→∞. The experiments related with ballistic phonons or electrons demonstrate the existence of the wave motion on the lattice scale or on the electron gas scale.

$$\frac{\partial^2 T}{\partial t^2} = \frac{D_T}{\tau}\nabla^2 T \tag{9}$$

For the systems with very long thermal history Eq. (7) is time symmetric with of course no arrow of time. For Eq. (9) shape does not change when t→ -t.

In Eq. (7) we define:

$$v = \left(\frac{D_T}{\tau}\right), \tag{10}$$

Velocity of thermal wave propagation, and

$$\lambda = \nu\tau \tag{11}$$

Where λ is defines the mean free path of heat carriers. With formula (10), the Eq. (7) can be rewritten as:

$$\frac{1}{\nu^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau\nu^2}\frac{\partial T}{\partial t} = \nabla^2 T \tag{12}$$

The equation

$$\frac{1}{\nu^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{D}\frac{\partial T}{\partial t} = \nabla^2 T$$

is the hyperbolic partial differential equation on one hand, and Fourier equation on the other hand

$$\frac{1}{D}\frac{\partial T}{\partial t} = \nabla^2 T \tag{13}$$

and Schrodinger equation

$$i\hbar\frac{\partial \psi}{\partial \mathrm{t}} = \nabla^2 \Psi \tag{14}$$

are both parabolic equations. Using substitution

$$t \leftrightarrow it, \;\; \Psi \leftrightarrow T \tag{15}$$

Fourier Eq. (13) can be rewritten as:

$$i\hbar\frac{\partial \Psi}{\partial t} = -D\hbar\nabla^2 \Psi \tag{16}$$

Using comparison with Schrodinger equation, we get:

$$D_T\hbar = \frac{\hbar^2}{2m} \tag{17}$$

$$\Longrightarrow \;\; D_T = \frac{\hbar}{2m} \tag{18}$$

If we consider $D_T=\tau v^2$ in Eq. (10), we obtain from Eq. (18)

$$\tau = \frac{\hbar}{2m\nu_{\hbar}^2} \tag{19}$$

The formula (19) describes the τ for the quantum thermal processes.

Starting with Schrodinger equation for a particle with mass m in potential V:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \tag{20}$$

Substituting (15), we get:

$$\hbar\frac{\partial T}{\partial t} = \frac{\hbar^2}{2m}\nabla^2 \mathrm{T} - \mathrm{VT} \tag{21}$$

$$\Longrightarrow \quad \frac{\partial T}{\partial t} = \frac{\hbar}{2m}\nabla^2 T - \frac{V}{\hbar}T \tag{22}$$

In Eq. (22) τ=o, we get:

$$\tau\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + \frac{V}{\hbar}T = \frac{\hbar}{2m}\nabla^2 T \tag{23}$$

and for τ≠0, we get:

$$\tau = \frac{\hbar}{2mv^2}$$

We take relaxation time τ as real constant in Eq. (23), we get:

$$\frac{1}{v^2}\frac{\partial^2 T}{\partial t^2} + \frac{2m}{\hbar}\frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2}T = \nabla^2 T \tag{24}$$

With substitution of Eq. (11) in Eq. (23), we get:

$$i\hbar\frac{\partial \Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\nabla^2\Psi - \tau\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{24}$$

The new relaxation term

$$\tau\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{25}$$

describes the interaction of the particle with mass m with space-time.

The relaxation time τ can be calculated as:

$$\tau^{-1} = (\tau_{e-p}^{-1} + \cdots + \tau_{Planck}^{-1} \tag{26}$$

Where $\tau_{e\text{-}p}$ denotes the scattering of the particle m on the electron-positron pair ($\tau_{e\text{-}p}$~10^{-17}S). The shortest relaxation time τ_{Planck} is the Planck time (τ_{Planck}~10^{-43}S).

From Eq.(26) we conclude that $\tau \approx \tau_{Planck}$, and Eq. (24) can be rewritten as:

$$i\hbar\frac{\partial \Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\nabla^2\Psi - \tau_{Planck}\hbar\frac{\partial^2\Psi}{\partial t^2} \tag{27}$$

Where,

$$\tau_{Planck} = \frac{1}{2}\left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \frac{\hbar}{2M_p c^2} \tag{28}$$

In Eq. (28) M_p is the planck mass.

Using the value of Eq. (28) in Eq. (27), we get:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi - \frac{\hbar^2}{2M_p}\nabla^2\Psi + \frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_p c^2}\frac{\partial^2\Psi}{\partial t^2} \tag{29}$$

Last two terms in the Eq. (29) are special because they can be defined as the *Bhomian Pilot wave*

$$\frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{30}$$

$$\Longrightarrow \nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{31}$$

We can observe that pilot wave Ψ does not depend on the mass of the particle.

Using the postulate from Eq. (31) in Eq. (29), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi - \frac{\hbar^2}{2M_p}\nabla^2\Psi \tag{32}$$

$$\Longrightarrow \text{simultaneously } \left\{\frac{\hbar^2}{2M_p}\nabla^2\Psi - \frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} = 0\right. \tag{33}$$

We can write Eq. (24) in operator form i.e.

$$\hat{E} = \frac{\hat{p}}{2m} + \frac{1}{2M_pc^2}\hat{E}^2 \tag{34}$$

Where $\hat{E}$ and $\hat{p}$ are operators for energy and momentum of the particle with mass m.

- Eq. (34) is the new dispersion relation for quantum particle with mass m
- From Eq. (24) we conclude that Schrodinger quantum mechanics is valid for particles with $m \ll M_p$.
- Pilot wave exists independent to the mass of the particle.

For the particle with $m \ll M_p$ Eq, (32) takes the form of:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \tag{35}$$

For the case $m \approx M_p$ Eq. (32) becomes,

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M_p}\nabla^2\Psi + V\Psi \tag{36}$$

but, using the case $m \approx M_p$ in Eq. (33), we get:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} + V\Psi \tag{37}$$

$$\Longrightarrow \frac{\hbar^2}{2M_pc^2}\frac{\partial^2\Psi}{\partial t^2} + i\hbar\frac{\partial\Psi}{\partial t} - V\Psi = 0 \tag{38}$$

For Eq. (38) we look for the solution in the form of:

$$\Psi(x,t) = e^{-i\omega}u(x) \tag{39}$$

Substituting the (39) in Eq. (38), we get:

$$\frac{\hbar^2}{2M_pc^2}\omega^2 - \omega\hbar + V(x) = 0 \tag{40}$$

With the solution,

For $\frac{M_pc^2}{2} > V$,

$$\omega_1 = -\frac{M_pc^2 + M_pc^2\sqrt{1-\frac{2V}{M_pc^2}}}{\hbar}$$

$$\omega_2 = -\frac{M_pc^2 - M_pc^2\sqrt{1-\frac{2V}{M_pc^2}}}{\hbar} \tag{41}$$

For $\frac{M_pc^2}{2} < V$,

$$\omega_1 = -\frac{M_pc^2 + iM_pc^2\sqrt{\frac{2V}{M_pc^2}-1}}{\hbar}$$

$$\omega_2 = -\frac{M_pc^2 - iM_pc^2\sqrt{\frac{2V}{M_pc^2}-1}}{\hbar} \tag{42}$$

Both (41) and (42) describe the "string oscillation", formula (30) damped oscillation, and formula (31) over damped string oscillation.

For elementary particle physics, the internal energy $M_p c^2$ is the maximum energy per particle in the universe.
In that case we can argue that the solution (41) is the valid solution[7].

For $\frac{M_pc^2}{2} < V$, we get:

$$\omega_1 = \frac{2M_pc^2}{\hbar}$$

$$\omega_2 = \frac{V}{\hbar} \tag{43}$$

The ω_1 represents Planck frequency $\omega_1 = \tau_p^{-1}$ and ω_2 is the frequency for Brain Waves.

Figure α represents ω_1 as the function of the ratio

$$\frac{2V}{M_p c^2} \tag{44}$$

We can observe from Fiqure α, for the potential energy V≈ 10^{-15}eV angular frequency of the brain waves is of the order 10Hz.

Considering that M_p equals to the mass of human neuron (10^5g) both Eqs. (30) and (31) describe the human neuron oscillations or emission of brain waves.

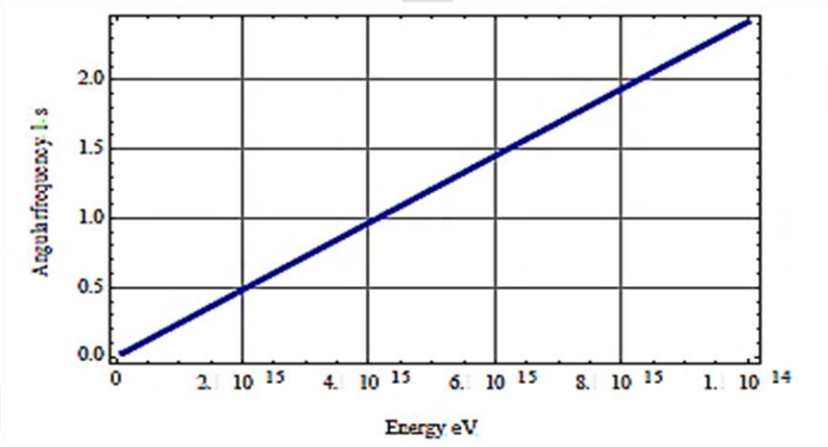

Figure α. Angular frequency as the function of energy.

3. CONCLUSION

In this paper we contemplated the relationship between 'Brain waves' and the 'Bohmian Mechanics', and we did it in order to imply an indirect relationship between the "Consciousness" and 'Physics'. We have seen that the brain waves indeed do act as solutions to the modified Schrodinger equation, thus giving a link between the brain waves and Quantum mechanics. We know that consciousness has the fundamental property i.e. the memory, we have shown link between the thermal memories of the quantum system by Generalized Fourier equation.

In a nutshell we implied relationship between Consciousness and Physics by showing a link between brain waves and quantum mechanics (i.e. link between thermal memory of a system and memory of human brain).

Indeed we humans are not yet at the level of understanding the consciousness, but as we keep contemplating it, we discover its very counterintuitive properties, and now we are a little closer towards the greatest leap i.e. consciousness is not just a subject of neuroscience, rather it is the new doctrine emerging from the shadows of Bio-physics.

REFERENCES

[1] Dürr, D.; Teufel, S. *Bohmian Mechanics: The Physics and Mathematics of Quantum Theory;*Springer:Berlin/Heidelberg, Germany, 2009.

[2] Bacciagaluppi, G.; Valentini, A. *Quantum Theory at the Crossroads Reconsidering the 1927 Solvay Confference;*Cambridge University Press: Cambridge, UK, 1993.

[3] Bohm, D,; Hiely, B.J. *The Undivided: An Ontological Interpretation of Quantum Theory;* Roueledge & Kegan Paul: London, UK, 1993.

[4] Bohm, D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. *Phys. Rev*. 1952, 85, 166-193.

[5] M. Kozlowski, J. Marciak-Kozlowska, The time arrow in a Planck gas, Foundations of Physics Letters, 10, p. 295, (1997).

[6] M. Kozlowski, J. Marciak-Kozlowska, Kelin-Gordon thermal equation for Planck Gas, Foundations of Physics Letters, 12, p. 93, (1999).

[7] J. Marciak-Kozlowska, M. Kozlowski, From infinity to infinity and beyond, Nova science publishers, 2014.

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